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UNITED STATES DEPARTMENT OF THE INTERIOR
GEOLOGICAL SURVEY

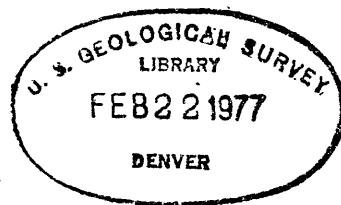
A computer program to calculate
the resistivity and induced polarization response
for a three-dimensional body
in the presence of buried electrodes

by

Jeffrey J. Daniels

Open-File Report 77-153

1976



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reviewed for conformity with U.S. Geological Survey
standards and nomenclature.

A computer program to calculate
the resistivity and induced polarization response
for a three-dimensional body
in the presence of buried electrodes

by Jeffrey J. Daniels
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Abstract

Three-dimensional induced polarization and resistivity modeling for buried electrode configurations can be achieved by adapting surface integral techniques for surface electrode configurations to buried electrodes. Modification of the surface technique is accomplished by considering the additional mathematical terms required to express the changes in the electrical potential and geometry caused by placing the source and receiver electrodes below the surface.

This report presents a listing of a computer program to calculate the resistivity and induced polarization response from a three-dimensional body for buried electrode configurations. The program is designed to calculate the response for the following electrode configurations: (1) hole-to-surface array with a buried bipole source and a surface bipole receiver, (2) hole-to-surface array with a buried pole source and a surface bipole receiver, (3) hole-to-hole array with a buried, fixed pole source and a moving bipole receiver, (4) surface-to-hole array with a fixed pole source on the surface and a moving bipole receiver in the borehole, (5) hole-to-hole array with

a buried, fixed bipole source and a buried, moving bipole receiver, (6) hole-to-hole array with a buried, moving bipole source and a buried, moving bipole receiver, and (7) single-hole, buried bipole-bipole array. Input and output examples are given for each of the arrays.

Introduction

A computer program was developed to calculate the theoretical apparent resistivity and apparent polarizability response for electrodes buried beneath the earth's surface in the presence of an arbitrarily shaped three-dimensional body. The program generates the response for three-dimensional ellipsoids of revolution for the following electrode configurations: (1) hole-to-surface with a buried bipole source and surface bipole receiver, (2) hole-to-surface with a buried pole source and a surface bipole receiver, (3) hole-to-hole array with a buried, fixed pole source and a moving bipole receiver, (4) surface-to-hole array with a fixed pole source on the surface and a moving bipole receiver in the borehole, (5) hole-to-hole array with a buried, fixed bipole source and buried, moving bipole receiver, (6) hole-to-hole array with a buried, moving bipole receiver, and (7) single-hole, buried bipole-bipole array. These arrays are illustrated in fig. 1 (AR1-AR7), and input and output examples for these are given in appendix A. The computer program is a modification of Barnett's (1972) three-dimensional IP modeling method for surface configurations to buried electrode configurations.

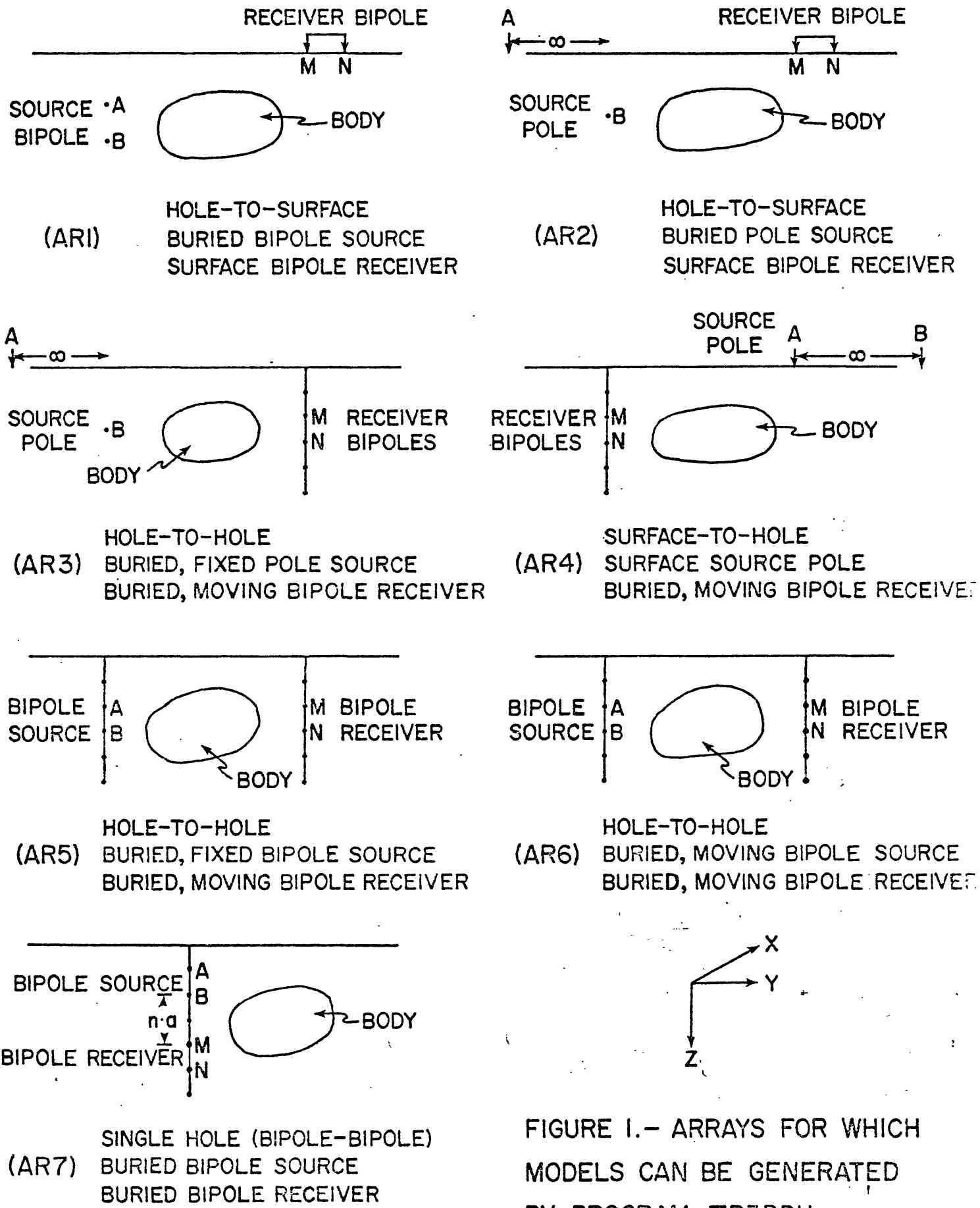


FIGURE I.- ARRAYS FOR WHICH MODELS CAN BE GENERATED BY PROGRAM IP3DDH.

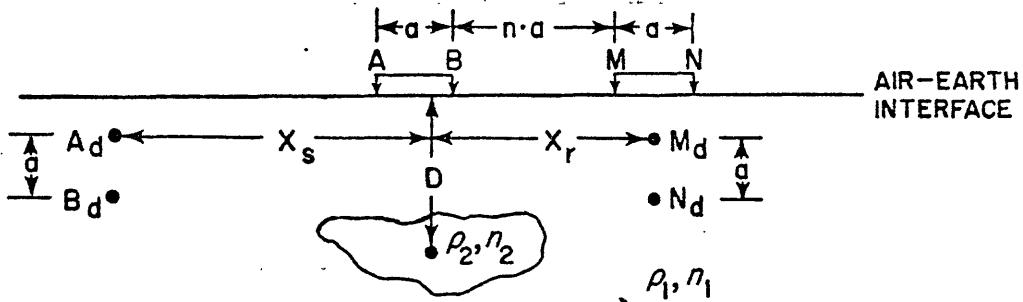
Hole-to-hole measurements are made by placing a current source pole or bipole in a borehole and a pole or bipole potential receiver in an adjacent borehole. The potential difference caused by the source is measured at discrete points in the receiver borehole. The source can be held at a stationary position (fixed source) for all of the potential measurements or the source can be moved when each potential measurement is made (moving source). A recent publication by Scott, et al. (1975) shows an application of hole-to-hole resistivity, induced polarization, and seismic measurements.

Hole-to-surface measurements are made by placing a pole or bipole source down a borehole and making surface bipole measurements radially away from the source hole. Theoretical studies of surface potentials due to inhole current sources have been described by Merkel (1971), Merkel and Alexander (1971), and Snyder and Merkel (1973).

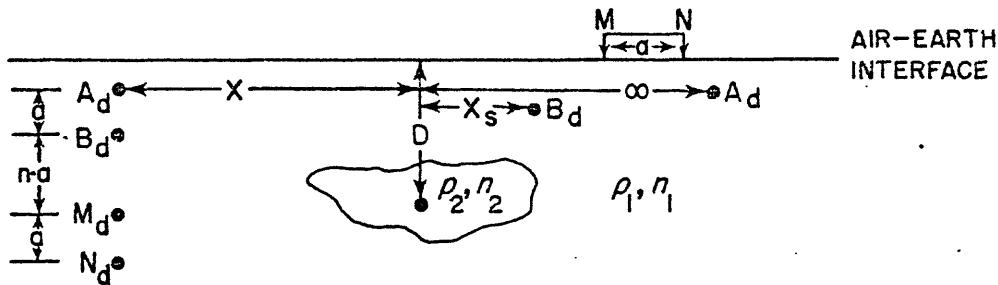
Wide-spaced single-hole arrays use a pole or bipole borehole source and a pole or bipole potential receiver in the same borehole. Several different source-receiver spacings can be used making a set of measurements similar to those made with a conventional surface dipole-dipole array. These measurements require only one borehole and can be made with a wireline device.

Theory

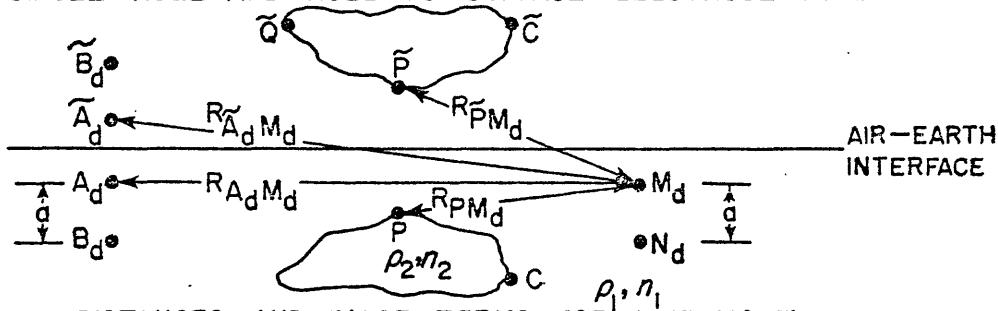
Figure 2 shows the combination of electrodes used in this study. The surface of the three-dimensional body is indicated by the letter "C", whereas the surface of the body's image in the upper halfspace is



ELECTRODE CONFIGURATIONS FOR HOLE-TO-HOLE AND SURFACE ARRAYS



SINGLE HOLE AND HOLE-TO-SURFACE ELECTRODE ARRAY



DISTANCES AND IMAGE TERMS FOR THE MODEL

EXPLANATION OF SYMBOLS :

ρ_1 = RESISTIVITY OF THE MEDIUM SURROUNDING THE BODY

ρ_2 = RESISTIVITY OF THE BODY

n_1 = INTRINSIC, INDUCED POLARIZATION OF THE MEDIUM SURROUNDING THE BODY

n_2 = INTRINSIC, INDUCED POLARIZATION OF THE BODY

A_d, B_d ARE THE BURIED SOURCE ELECTRODES

M_d, N_d ARE THE BURIED RECEIVER ELECTRODES

A, B ARE THE SURFACE SOURCE ELECTRODES

M, N ARE THE SURFACE RECEIVER ELECTRODES

\tilde{A}_d, \tilde{B}_d ARE THE REFLECTIONS OF THE BURIED SOURCE ELECTRODES

X, X_s, X_r ARE HORIZONTAL DISTANCES

$Q, \tilde{P}, \tilde{C}, P, C$ ARE POINTS REFERRED TO IN THE EQUATIONS IN THE TEXT

FIGURE 2 — THREE DIMENSIONAL BODY AND ELECTRODE POSITIONS FOR THE MODEL USED IN THIS STUDY

designated by the symbol " \tilde{C} ". All depth and distance used are normalized by the receiver-bipole spacing "a". A bipole is a finitely spaced electrode pair. The following is a brief outline of the theory. A more complete outline is given in appendix B and in papers by Barnett (1972) and Daniels (1977).

For a conventional "bipole-bipole" surface configuration, where either the source and(or) the receiver is on the surface, the equation for apparent resistivity is

$$\rho_a = \frac{2\pi}{\frac{1}{R_{AM}} + \frac{1}{R_{AN}} - \frac{1}{R_{BM}} + \frac{1}{R_{BN}}} \frac{(U_M - U_N)}{I} \quad (1)$$

where R_{AM} , R_{AN} , R_{BM} , and R_{BN} are the respective distances between electrodes A, M, N, and B; U_M is the electrical potential at point M; U_N is the electrical potential at point N; and I is the electric current. When both the source and receiver are buried, the apparent resistivity formula becomes

$$\rho_a = \frac{4\pi}{\frac{1}{R_{Ad}^M d} - \frac{1}{R_{Ad}^N d} - \frac{1}{R_{Bd}^M d} + \frac{1}{R_{Bd}^N d} + \frac{i}{R_{Ad}^M d} - \frac{1}{R_{Ad}^N d} - \frac{1}{R_{Bd}^M d} + \frac{1}{R_{Bd}^N d}} \cdot \left(\frac{(U_{Md} - U_{Nd})}{I} \right) \quad (2)$$

Barnett has shown that the expression for apparent polarizability can be expressed in a convenient computational form as

$$\frac{n_a - n_1}{n_2 - n_1} = \frac{(1 - K^2)}{2\rho_a} \frac{\partial \rho_a}{\partial K} \quad (3)$$

where η_a is the apparent polarizability, η_2 is the polarizability of the body, η_1 is the polarizability of the surrounding medium,

$K = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}$ = resistivity reflection coefficient, ρ_a is the apparent resistivity, ρ_2 is the resistivity of the body, and ρ_1 is the

resistivity of the surrounding medium. $\frac{\eta_a}{\eta_2} \times 100$ is the same as $B_2 (\%)$ presented by Snyder and Merkel (1973).

The expression for the potential U_{M_d} at a point M_d due to a current source at A_d has been written by Barnett (1972) as

$$U_{M_d} = \frac{\rho_1 I}{4\pi} \left(\frac{1}{R_{A_d M_d}} + \frac{1}{R_{A_d^* M_d}} \right) + \frac{1}{4\pi} \left\{ \int_C \sigma_p \frac{1}{R_{p M_d}} dC + \int_{C^*} \sigma_{p^*} \frac{1}{R_{p M_d}} dC^* \right\} \quad (4)$$

where σ_p and σ_{p^*} represent the equivalent surface charge density distributions at points p and p^* on the body, C , and its image, C^* , and $R_{A_d M_d}$, $R_{A_d^* M_d}$, $R_{p M_d}$, and $R_{p M_d^*}$ are the distances shown in fig. 2. The

computational form for this equation is

$$U_{M_d} = \frac{\rho_1 I}{4\pi} \left\{ \frac{1}{R_{A_d M_d}} + \frac{1}{R_{A_d^* M_d}} + \sum_{i=1}^N S_i(A_d) H_i(M_d) \right\} \quad (5)$$

where the surface of the body is divided into N triangular facets and $S_i(A_d)$ and $H_i(M_d)$ are the source and receiver response contributions from each individual body-facet (Barnett, 1972). If the current source

is at A_d , and a current sink is at B_d , the potential difference between points M_d and N_d becomes

$$U_{M_d} - U_{N_d} = \frac{\rho_1 I}{4\pi} \left\{ \frac{1}{R_{A_d M_d}} + \frac{1}{R_{A_d M_d}^*} - \frac{1}{R_{B_d M_d}} - \frac{1}{R_{B_d M_d}^*} - \frac{1}{R_{A_d N_d}} - \frac{1}{R_{A_d N_d}^*} + \frac{1}{R_{B_d N_d}} + \frac{1}{R_{B_d N_d}^*} + \sum_{i=1}^N [(S_i(A_d) - S_i(B_d)) (H_i(M_d) - H_i(N_d))] \right\} \quad (6)$$

The derivative of the potential difference with respect to the resistivity reflection coefficient is

$$\frac{\partial U_{M_d}}{\partial K} - \frac{\partial U_{N_d}}{\partial K} = \frac{\rho_1 I N}{2\pi} \sum_{i=1}^N [(T_i(A_d) - T_i(B_d)) (H_i(M_d) - H_i(N_d))] \quad (7)$$

where $T_i(A_d) = \frac{\partial S_i(A_d)}{\partial K}$ (Barrett, 1972). Substituting equations (6) and (7) into equations (2) and (3) respectively, the final computational form for the apparent resistivity normalized with respect to ρ_1 is

$$\frac{\rho_a}{\rho_1} = 1 + \frac{\sum_{i=1}^N [(S_i(A_d) - S_i(B_d)) (H_i(M_d) - H_i(N_d))]}{\frac{1}{R_{A_d M_d}} + \frac{1}{R_{A_d M_d}^*} - \frac{1}{R_{B_d M_d}} - \frac{1}{R_{B_d M_d}^*} - \frac{1}{R_{A_d N_d}} - \frac{1}{R_{A_d N_d}^*} + \frac{1}{R_{B_d N_d}} + \frac{1}{R_{B_d N_d}^*}} \quad (8)$$

and the computational expression for apparent polarizability is

$$\frac{n_a + n_1}{n_2 + n_1} = \frac{(1 - K^2) \sum_{i=1}^N [(T_i(A_d) - T_i(B_d)) (H_i(M_d) - H_i(N_d))]}{\frac{1}{R_{A_d M_d}} + \frac{1}{R_{A_d M_d}^*} - \frac{1}{R_{B_d M_d}} - \frac{1}{R_{B_d M_d}^*} - \frac{1}{R_{A_d N_d}} - \frac{1}{R_{A_d N_d}^*} + \frac{1}{R_{B_d N_d}} + \frac{1}{R_{B_d N_d}^*}} \left(\frac{\rho_1}{\rho_a} \right) \quad (9)$$

A more complete development is given in appendix B.

Explanation of the Computer Program

The computer program which generates the three-dimensional body and calculates the response for the various arrays was developed on a Digital Equipment Corporation (DEC) PDP-10 computer.^{1/} The program outlined in fig. 3 is written in FORTRAN IV. The input and the output device is disk (input disk is specified as logical unit 13, output disk is logical unit 14), which is specified in "COMMON" and can be easily changed.

An ellipsoid of revolution, with 72 facets, is generated by the program when the x, y, and z half-width (a, b, and C) of the body are specified. The response for an arbitrarily shaped body may be calculated by eliminating the call to SUBROUTINE BODY3D and changing SUBROUTINE READ3D.

Input and output examples for the seven different arrays are given in appendix A. The sphere model used for these examples is shown in fig. 4. A listing of the computer program is given in appendix C. Computation time for one set of measurements is approximately 1 minute of CPU time on the PDP-10.

Since this is a one-of-a-kind computer program, it is impossible to absolutely verify the results. However, I have made a rough check with Snyder's and Merkel's (1973) hole-to-surface idealized sphere model.

^{1/} The use of brand names in this report is for descriptive purposes only and in no way constitutes endorsement by the U.S. Geological Survey.

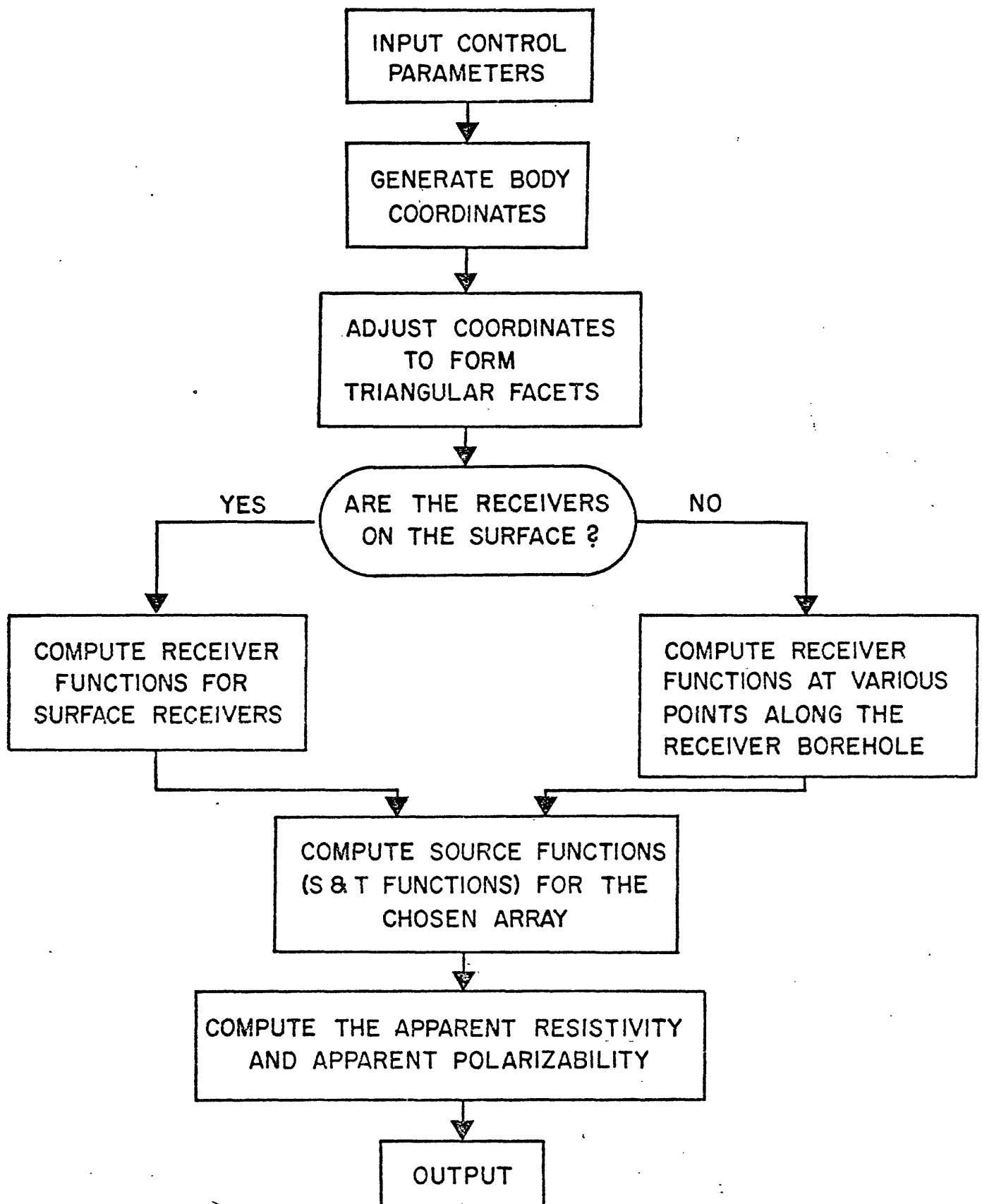
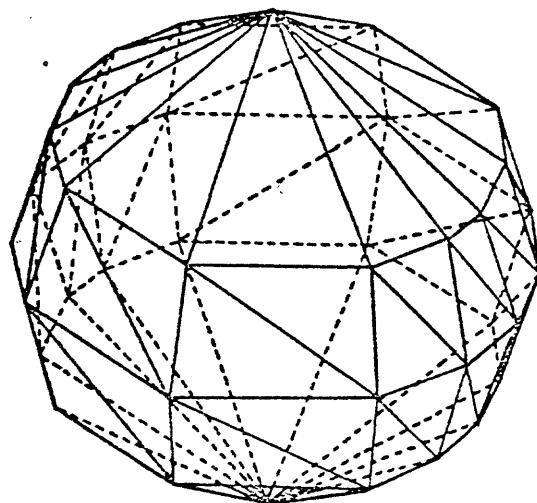


FIGURE 3.- COMPUTATIONAL FLOW OF IP3DDH.

SPHERE



X-DIAMETER = 1.0 UNIT
Y-DIAMETER = 1.0 UNIT.
Z-DIAMETER = 1.0 UNIT

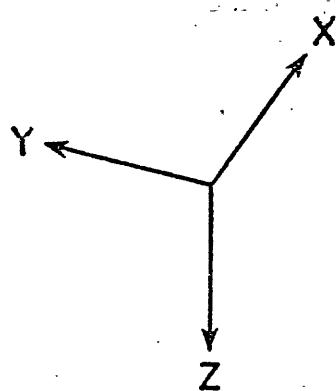


FIGURE 4.- THREE DIMENSIONAL MODEL USED FOR TEST CASES.

References

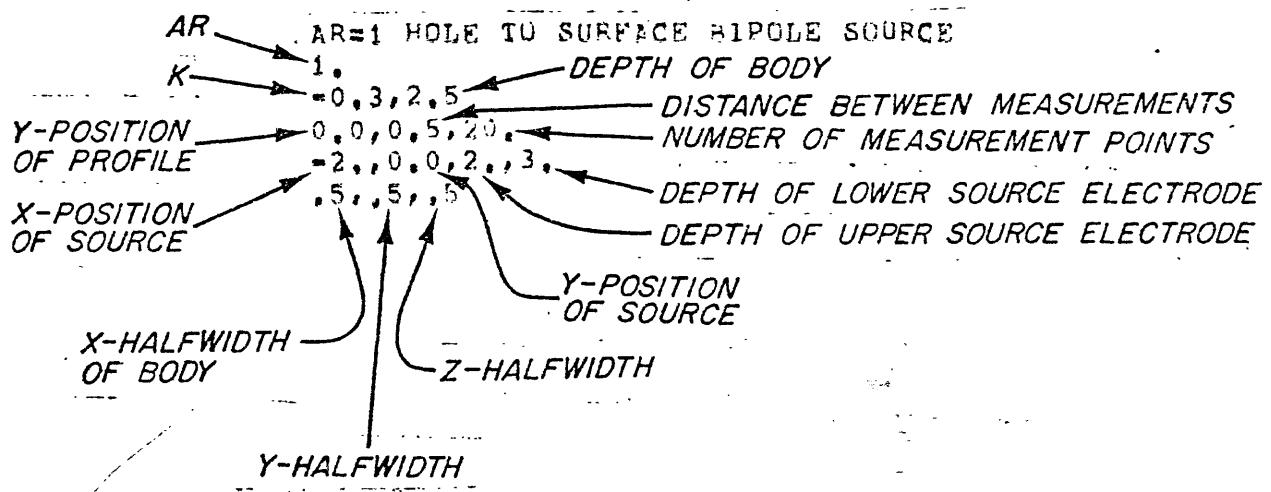
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- Scott, J. H., Daniels, J. J., Hasbrouck, W. P., and Guu, J. Y., 1975, Hole-to-hole geophysical measurement research for mineral exploration: Trans. 16th Ann. Logging Symp.
- Snyder, D. D., and Merkel, R. M., 1973, Analytic models for the interpretation of electrical surveys using buried current electrodes: Geophysics, v. 28, p. 513-529.

Appendix A

Input-output examples

HOLE-TO-SURFACE BURIED BIPOLE SOURCE SURFACE BIPOLE RECEIVER

INPUT DATA



OUTPUT DATA

3-DIMENSIONAL IP AND RESISTIVITY MODELLING PROGRAM

DATE: 24-JUN-76 TIME: 10:28 PAGE:

INPUT CONTROL PARAMETERS:

KVALU -0.30000
DEPTH 2.50
EXECU

DESCRIPTION OF MODEL: AR=1 HOLE TO SURFACE
BIPOLE SOURCE

Z-COORDINATE	(X,Y)-COORDINATES IN PAIRS				
TOP	-0.5000000	0.0000000	0.0000000		
CONTOUR 1	-0.2500000	0.2886751	0.3227486	0.1443376	0.4082483
		0.0000000	0.4330127	-0.1443376	0.4082483
-0.2886751	0.3227486	-0.4330127	0.0000000	-0.2886751	-0.3227486
-0.1443376	-0.4082483	0.0000000	-0.4330127	0.1443376	-0.4082483
0.2886751	-0.3227486	0.4330127	0.0000000		
CONTOUR 2	0.0000000	0.1666667	0.4714045	0.0000000	0.5000000
		-0.1666667	0.4714045	-0.3333333	0.3726780
-0.5000000	0.0000000	-0.3333333	-0.3726780	-0.1666667	-0.4714045
0.0000000	-0.5000000	0.1666667	-0.4714045	0.3333333	-0.3726780
0.5000000	0.0000000	0.3333333	0.3726780		
CONTOUR 3	0.2500000	0.0000000	0.4330127	-0.1443376	0.4082483
		-0.2886751	0.3227486	-0.4330127	0.0000000
-0.2886751	-0.3227486	-0.1443376	-0.4082483	0.0000000	-0.4330127
0.1443376	-0.4082483	0.2886751	-0.3227486	0.4330127	0.0000000
0.2886751	0.3227486	0.1443376	0.4082483		
BOTTOM	0.5000000	0.0000000	0.0000000	0.0000000	

3-DIMENSIONAL IP AND RESISTIVITY MODELLING PROGRAM

DATE: 24-JUN-76

TIME: 10:28

DESCRIPTION OF MODEL: AR=1 HOLE TO SURFACE
BIPOLE SOURCEREFLECTION COEFF. K = -0.30000
DEPTH = 2.50

 BURIED BIPOLE SOURCE, SURFACE BIPOLE RECEIVER

UPPER SOURCE= 2.000
 LOWER SOURCE= 3.000
 X-SOURCE= -2.000
 Y-SOURCE= 0.000

Y-PROFILE= 0.000

X-RECEIVER POSITION	APPARENT RESISTIVITY	APPARENT POLARIZABILITY
-10.500	.998608E+00	.219814E-02
-10.000	.998663E+00	.210880E-02
-9.500	.998720E+00	.201754E-02
-9.000	.998778E+00	.192456E-02
-8.500	.998837E+00	.183029E-02
-8.000	.998897E+00	.173512E-02
-7.500	.998957E+00	.163985E-02
-7.000	.999016E+00	.154581E-02
-6.500	.999073E+00	.145493E-02
-6.000	.999127E+00	.137024E-02
-5.500	.999173E+00	.129664E-02
-5.000	.999207E+00	.124240E-02
-4.500	.999220E+00	.122253E-02
-4.000	.999193E+00	.126335E-02
-3.500	.999078E+00	.144302E-02
-3.000	.998726E+00	.199481E-02
-2.500	.997398E+00	.407686E-02
-2.000	.170141E+39	.100000E+01
-1.500	.100367E+01	-.571242E-02
-1.000	.100213E+01	-.332900E-02
-0.500	.100109E+01	-,170120E-02
0.000	.999446E+00	.859055E-03
0.500	.997121E+00	.450470E-02
1.000	.994683E+00	.835363E-02
1.500	.992815E+00	.113208E-01
2.000	.991791E+00	.129559E-01
2.500	.991465E+00	.134809E-01
3.000	.991581E+00	.132991E-01

3,500	991964E+00	,126930E-01
4,000	992430E+00	,119558E-01
4,500	992871E+00	,112558E-01
5,000	993300E+00	,105759E-01
5,500	993694E+00	,995339E-02
6,000	994048E+00	,939359E-02
6,500	994365E+00	,889414E-02
7,000	994647E+00	,844951E-02
7,500	994899E+00	,805345E-02
8,000	995125E+00	,769987E-02

HOLE-TO-HOLE
BURIED, FIXED POLE SOURCE
BURIED, MOVING BIPOLE RECEIVER

INPUT DATA

AR=3 HOLE TO HOLE FIXED SOURCE POLE
3,
-0.,3,1.5
2.,0.,0.,0.,5,10.
-2.,0.,0,2.
.5,,5,.5

OUTPUT DATA

3-DIMENSIONAL IP AND RESISTIVITY MODELLING PROGRAM

DATE: 24-JUN-76

TIME: 11:28

PAGE:

INPUT CONTROL PARAMETERS:

KVALU -0.30000
DEPTH 1.50
EXECU

DESCRIPTION OF MODEL: AR=3 HOLE TO HOLE
FIXED SOURCE POLE

Z-COORDINATE	(X,Y)-COORDINATES IN PAIRS	-----
TOP -0.5000000	0.0000000 0.0000000	
CONTOUR 1 -0.2500000	0.2886751 0.3227486 0.1443376 0.4082483	
	0.0000000 0.4330127 -0.1443376 0.4082483	
-0.2886751 0.3227486 -0.4330127 0.0000000	-0.2886751 -0.3227486 0.1443376 -0.4082483	
-0.1443376 -0.4082483 0.0000000 -0.4330127	0.1443376 -0.4082483 0.4330127 0.0000000	
0.2886751 -0.3227486 0.4330127 0.0000000		
CONTOUR 2 0.0000000	0.1666667 0.4714045 0.0000000 0.5000000	
	-0.1666667 0.4714045 -0.3333333 0.3726780	
-0.5000000 0.0000000 -0.3333333 -0.3726780	-0.1666667 -0.4714045 0.3333333 -0.3726780	
0.0000000 -0.5000000 0.1666667 -0.4714045	0.3333333 -0.3726780 0.4714045 0.0000000	
0.5000000 0.0000000 0.3333333 0.3726780		
CONTOUR 3 0.2500000	0.0000000 0.4330127 -0.1443376 0.4082483	
	-0.2886751 0.3227486 -0.4330127 0.0000000	
-0.2886751 -0.3227486 -0.1443376 -0.4082483	0.0000000 -0.4330127 0.1443376 0.4082483	
0.1443376 -0.4082483 0.2886751 -0.3227486	0.4330127 0.0000000 0.4082483 -0.4330127	
0.2886751 0.3227486 0.1443376 0.4082483		
BOTTOM 0.5000000	0.0000000 0.0000000	

3-DIMENSIONAL IP AND RESISTIVITY MODELLING PROGRAM

DATE: 24-JUN-76

TIME: 11:28

DESCRIPTION OF MODEL: AR=3 HOLE TO HOLE
FIXED SOURCE POLE

REFLECTION COEFF, K = -0.30000
DEPTH = 1.50

BURIED POLE SOURCE, BURIED BIPOLE RECEIVER

UPPER SOURCE= 0.000
LOWER SOURCE= 2.000
X-SOURCE= -2.000
Y-SOURCE= 0.000

X-RECEIVER= 2.000
Y-RECEIVER= 0.000

RECEIVER APPARENT APPARENT
DEPTH RESISTIVITY POLARIZABILITY

1.000	.988036E+00	.192324E-01
1.500	.100980E+01	-.154555E-01
2.000	.102068E+01	-.322144E-01
2.500	.102042E+01	-.318101E-01
3.000	.101539E+01	-.240850E-01
3.500	.101041E+01	-.163674E-01
4.000	.100689E+01	-.108710E-01
4.500	.100465E+01	-.736094E-02
5.000	.100325E+01	-.515087E-02
5.500	.100236E+01	-.373968E-02
6.000	.100178E+01	-.282628E-02
6.500	.100139E+01	-.220694E-02
7.000	.100111E+01	-.176749E-02
7.500	.100091E+01	-.144696E-02
8.000	.100076E+01	-.121151E-02
8.500	.100064E+01	-.103006E-02
9.000	.100055E+01	-.888375E-03
9.500	.100048E+01	-.775617E-03

HOLE-TO-HOLE
BURIED, FIXED BIPOLE SOURCE
BURIED, MOVING BIPOLE RECEIVER

INPUT DATA

R=5 HOLE-TO-HOLE FIXED SOURCE
5.
-0,3,1,5
2,,0,0,,25,11,
-2,,0,0,2,,3,
,5,,5,,5

OUTPUT DATA

3-DIMENSIONAL IP AND RESISTIVITY MODELLING PROGRAM

DATE: 24-JUN-76 TIME: 12:06 PAGE:

INPUT CONTROL PARAMETERS:

KVALU -0.30000
DEPTH 1.50
EXECU

DESCRIPTION OF MODEL: R=5 HOLE-TO-HOLE
FIXED SOURCE

Z-COORDINATE	(X,Y)-COORDINATES IN PAIRS				
TOP	-0.5000000	0.0000000	0.0000000		
CONTOUR 1	-0.2500000	0.2886751	0.3227486	0.1443376	0.4082483
		0.0000000	0.4330127	-0.1443376	0.4082483
-0.2886751	0.3227486	-0.4330127	0.0000000	-0.2886751	-0.3227486
-0.1443376	-0.4082483	0.0000000	-0.4330127	0.1443376	-0.4082483
0.2886751	-0.3227486	0.4330127	0.0000000		
CONTOUR 2	0.0000000	0.1666667	0.4714045	0.0000000	0.5000000
		-0.1666667	0.4714045	-0.3333333	0.3726780
-0.5000000	0.0000000	-0.3333333	-0.3726780	-0.1666667	-0.4714045
0.0000000	-0.5000000	0.1666667	-0.4714045	0.3333333	-0.3726780
0.5000000	0.0000000	0.3333333	0.3726780		
CONTOUR 3	0.2500000	0.0000000	0.4330127	-0.1443376	0.4082483
		-0.2886751	0.3227486	-0.4330127	0.0000000
-0.2886751	-0.3227486	-0.1443376	-0.4082483	0.0000000	-0.4330127
0.1443376	-0.4082483	0.2886751	-0.3227486	0.4330127	0.0000000
0.2886751	0.3227486	0.1443376	0.4082483		
BOTTOM	0.5000000	0.0000000	0.0000000	0.0000000	

3-DIMENSIONAL IP AND RESISTIVITY MODELLING PROGRAM

DATE: 24-JUN-76

TIME: 12:06

DESCRIPTION OF MODEL: R=5 HOLE-TO-HOLE
FIXED SOURCE

REFLECTION COEFF. K = -0.30000

DEPTH = 1.50

 BURIED BIPOLE FIXED SOURCE, BURIEDBIPOLE RECEIVER

UPPER SOURCE= 2.000

LOWER SOURCE= 3.000

X-SOURCE= -2.000

Y-SOURCE= 0.000

X-RECEIVER= 2.000

Y-RECEIVER= 0.000

RECEIVER DEPTH	APPARENT RESISTIVITY	APPARENT POLARIZABILITY
----------------	----------------------	-------------------------

0.750	.978771E+00	.341950E-01
1.000	.983506E+00	.263412E-01
1.250	.989042E+00	.172621E-01
1.500	.994820E+00	.790791E-02
1.750	.100023E+01	-.729730E-03
2.000	.100475E+01	-.787284E-02
2.250	.100811E+01	-.131193E-01
2.500	.101030E+01	-.165037E-01
2.750	.101152E+01	-.183671E-01
3.000	.101206E+01	-.191650E-01
3.250	.101220E+01	-.193646E-01
3.500	.101219E+01	-.193361E-01
3.750	.101220E+01	-.193458E-01
4.000	.101236E+01	-.196034E-01
4.250	.101281E+01	-.203079E-01
4.500	.101364E+01	-.216319E-01
4.750	.101511E+01	-.239403E-01
5.000	.101763E+01	-.279027E-01
5.250	.102217E+01	-.349734E-01
5.500	.103193E+01	-.499566E-01
5.750	.106326E+01	-.961867E-01
6.000	.588944E-01	.258781E+02
6.250	.947078E+00	.906410E-01
6.500	.973251E+00	.446628E-01
6.750	.982209E+00	.294908E-01
7.000	.986483E+00	.220189E-01
7.250	.989339E+00	.176170E-01

7.500	.991082E+00	.147418E-01
7.750	.992305E+00	.127329E-01
8.000	.993204E+00	.112611E-01
8.250	.993888E+00	.101441E-01
8.500	.994423E+00	.927308E-02
8.750	.994851E+00	.857914E-02
9.000	.995199E+00	.801664E-02
9.250	.995487E+00	.755420E-02
9.500	.995727E+00	.716952E-02
9.750	.995929E+00	.684643E-02
10.000	.996102E+00	.657294E-02
10.250	.996250E+00	.633982E-02
10.500	.996377E+00	.614011E-02
10.750	.000000E+00	.000000E+00
11.000	.000000E+00	.000000E+00
11.250	.000000E+00	.000000E+00

HOLE-TO-HOLE
BURIED, MOVING BIPOLE SOURCE
BURIED, MOVING BIPOLE RECEIVER

INPUT DATA

AR=6 HOLE-TO-HOLE MOVING SOURCE
6.
-0,3,1.5
2.,0,0,0,25,11,
-2,,0,,0
,5,,5,,5

OUTPUT DATA

3-DIMENSIONAL IP AND RESISTIVITY MODELLING PROGRAM

DATE: 24-JUN-76 TIME: 12:14 PAGE:

INPUT CONTROL PARAMETERS:

KVALU -0.30000
DEPTH 1.50
EXECU

DESCRIPTION OF MODEL: AR=6 HOLE-TO-HOLE
MOVING SOURCE

Z-COORDINATE	(X, Y)-COORDINATES IN PAIRS				
TOP	-0.5000000	0.0000000	0.0000000		
CONTOUR 1	-0.2500000	0.2886751	0.3227486	0.1443376	0.4082483
		0.0000000	0.4330127	-0.1443376	0.4082483
-0.2886751	0.3227486	-0.4330127	0.0000000	-0.2886751	-0.3227486
-0.1443376	-0.4082483	0.0000000	-0.4330127	0.1443376	-0.4082483
0.2886751	-0.3227486	0.4330127	0.0000000		
CONTOUR 2	0.0000000	0.1666667	0.4714045	0.0000000	0.5000000
		-0.1666667	0.4714045	-0.3333333	0.3726780
-0.5000000	0.0000000	-0.3333333	-0.3726780	-0.1666667	-0.4714045
0.0000000	-0.5000000	0.1666667	-0.4714045	0.3333333	-0.3726780
0.5000000	0.0000000	0.3333333	0.3726780		
CONTOUR 3	0.2500000	0.0000000	0.4330127	-0.1443376	0.4082483
		-0.2886751	0.3227486	-0.4330127	0.0000000
-0.2886751	-0.3227486	-0.1443376	-0.4082483	0.0000000	-0.4330127
0.1443376	-0.4082483	0.2886751	-0.3227486	0.4330127	0.0000000
0.2886751	0.3227486	0.1443376	0.4082483		
BOTTOM	0.5000000	0.0000000	0.0000000	0.0000000	

3-DIMENSIONAL IP AND RESISTIVITY MODELLING PROGRAM

DATE: 24-JUN-76 TIME: 12:14

DESCRIPTION OF MODEL: AR=6 HOLE-TO-HOLE
MOVING SOURCE

REFLECTION COEFF. K = -0.30000
DEPTH = 1.50

BURIED BIPOLE MOVING SOURCE, BURIED BIPOLE RECEIVER

X-SOURCE= -2.000
Y-SOURCE= 0.000

X-RECEIVER= 2.000
Y-RECEIVER= 0.000

RECEIVER APPARENT APPARENT
DEPTH RESISTIVITY POLARIZABILITY

0.750	.981031E+00	.301491E-01
1.000	.977376E+00	.362583E-01
1.250	.976248E+00	.381801E-01
1.500	.979436E+00	.328914E-01
1.750	.986924E+00	.205667E-01
2.000	.996536E+00	.503984E-02
2.250	.100509E+01	-.846306E-02
2.500	.101030E+01	-.165037E-01
2.750	.101183E+01	-.187483E-01
3.000	.101076E+01	-.169747E-01
3.250	.100853E+01	-.134282E-01
3.500	.100618E+01	-.972554E-02
3.750	.100422E+01	-.663548E-02
4.000	.100277E+01	-.435051E-02
4.250	.100178E+01	-.277940E-02
4.500	.100112E+01	-.174295E-02
4.750	.100069E+01	-.107910E-02
5.000	.100043E+01	-.659909E-03
5.250	.100026E+01	-.400701E-03
5.500	.100016E+01	-.240227E-03
5.750	.100010E+01	-.141944E-03
6.000	.100006E+01	-.827301E-04
6.250	.100003E+01	-.457493E-04
6.500	.100002E+01	-.234697E-04
6.750	.100001E+01	-.102049E-04
7.000	.100000E+01	-.247960E-05
7.250	.100000E+01	.185553E-05
7.500	.999999E+00	.413070E-05
7.750	.999998E+00	.517104E-05

8,000	.999998E+00	.548506E-05
8,250	.999998E+00	.538617E-05
8,500	.999998E+00	.506644E-05
8,750	.999998E+00	.464168E-05
9,000	.999998E+00	.418133E-05
9,250	.999998E+00	.372468E-05
9,500	.999998E+00	.329306E-05
9,750	.999999E+00	.289674E-05
10,000	.999999E+00	.253971E-05
10,250	.999999E+00	.222248E-05
10,500	.999999E+00	.194278E-05

SINGLE HOLE
MOVING BIPOLE SOURCE
MOVING BIPOLE RECEIVER

INPUT DATA

AR=7 SINGLE-HOLE N=3
7,
-0,3,3,5
-2,,0,0,.5,10,
3
.5,,5,,5

OUTPUT DATA

3-DIMENSIONAL IP AND RESISTIVITY MODELLING PROGRAM

DATE: 24-JUN-76 TIME: 12:27 PAGE:

INPUT CONTROL PARAMETERS:

KVALU -0.30000
DEPTH 3.50
EXECU

DESCRIPTION OF MODEL: AR=7 SINGLE-HOLE N=3

Z-COORDINATE (X,Y)-COORDINATES IN PAIRS -----

TOP	-0.5000000	0.0000000	0.0000000			
CONTOUR 1	-0.2500000	0.2886751	0.3227486	0.1443376	0.4082483	
		0.0000000	0.4330127	-0.1443376	0.4082483	
-0.2886751	0.3227486	-0.4330127	0.0000000	-0.2886751	0.3227486	
-0.1443376	-0.4082483	0.0000000	-0.4330127	0.1443376	-0.4082483	
0.2886751	-0.3227486	0.4330127	0.0000000			
CONTOUR 2	0.0000000	0.1666667	0.4714045	0.0000000	0.5000000	
		-0.1666667	0.4714045	-0.3333333	0.3726780	
-0.5000000	0.0000000	-0.3333333	-0.3726780	-0.1666667	-0.4714045	
0.0000000	-0.5000000	0.1666667	-0.4714045	0.3333333	-0.3726780	
0.5000000	0.0000000	0.3333333	0.3726780			
CONTOUR 3	0.2500000	0.0000000	0.4330127	-0.1443376	0.4082483	
		-0.2886751	0.3227486	-0.4330127	0.0000000	
-0.2886751	-0.3227486	-0.1443376	-0.4082483	0.0000000	-0.4330127	
0.1443376	-0.4082483	0.2886751	-0.3227486	0.4330127	0.0000000	
0.2886751	0.3227486	0.1443376	0.4082483			
BOTTOM	0.5000000	0.0000000	0.0000000	0.0000000		

3-DIMENSIONAL IP AND RESISTIVITY MODELLING PROGRAM

DATE: 24-JUN-76

TIME: 12:27

DESCRIPTION OF MODEL: AR=7 SINGLE-HOLE N=3

REFLECTION COEFF. K = -0.30000
DEPTH = 3.50

SINGLE HOLE,BIPOLE-BIPOLE

X-POSITION OF HOLE= -2.000
Y-POSITION OF HOLE= 0.000

RECEIVER DEPTH	APPARENT RESISTIVITY	APPARENT POLARIZABILITY
3.000	.997100E+00	.462375E-02
3.500	.997080E+00	.464685E-02
4.000	.997170E+00	.445965E-02
4.500	.997415E+00	.401191E-02
5.000	.997986E+00	.310014E-02
5.500	.998854E+00	.178910E-02
6.000	.999683E+00	.533146E-03
6.500	.100016E+01	-.218523E-03
7.000	.100028E+01	-.434588E-03
7.500	.100023E+01	-.371253E-03
8.000	.100055E+01	-.901999E-03
8.500	.100037E+01	-.612888E-03

HOLE-TO-SURFACE
BURIED POLE SOURCE
SURFACE BIPOLE RECEIVER

INPUT DATA

L AR=2 HOLE-TO-SURFACE SOURCE POLE
2,
-0,3,1.5
0,0,0.5,20,
-2,,0,0,2.5
.5,.5,.5

OUTPUT DATA

3-DIMENSIONAL IP AND RESISTIVITY MODELLING PROGRAM

DATE: 24-JUN-76

TIME: 10:36

PAGE:

INPUT CONTROL PARAMETERS:

KVALU -0,30000
DEPTH 1,50
EXECU

DESCRIPTION OF MODEL: AR=2 HOLE-TO-SURFACE
SOURCE POLE

Z-COORDINATE (X,Y)-COORDINATES IN PAIRS -----

TOP	-0,5000000	0,0000000	0,0000000			
CONTOUR 1	-0,2500000	0,2886751	0,3227486	0,1443376	0,4082483	
		0,0000000	0,4330127	-0,1443376	0,4082483	
-0,2886751	0,3227486	-0,4330127	0,0000000	-0,2886751	-0,3227486	
-0,1443376	-0,4082483	0,0000000	-0,4330127	0,1443376	-0,4082483	
0,2886751	-0,3227486	0,4330127	0,0000000			
CONTOUR 2	0,0000000	0,1666667	0,4714045	0,0000000	0,5000000	
		-0,1666667	0,4714045	-0,3333333	0,3726780	
-0,5000000	0,0000000	-0,3333333	-0,3726780	-0,1666667	-0,4714045	
0,0000000	-0,5000000	0,1666667	0,4714045	0,3333333	-0,3726780	
0,5000000	0,0000000	0,3333333	0,3726780			
CONTOUR 3	0,2500000	0,0000000	0,4330127	-0,1443376	0,4082483	
		-0,2886751	0,3227486	-0,4330127	0,0000000	
-0,2886751	-0,3227486	-0,1443376	-0,4082483	0,0000000	-0,4330127	
0,1443376	-0,4082483	0,2886751	-0,3227486	0,4330127	0,0000000	
0,2886751	0,3227486	0,1443376	0,4082483			
BOTTOM	0,5000000	0,0000000	0,0000000	0,0000000		

3-DIMENSIONAL IP AND RESISTIVITY MODELLING PROGRAM

DATE: 24-JUN-76

TIME: 10:36

DESCRIPTION OF MODEL: AR=2 HOLE-TO-SURFACE
SOURCE POLEREFLECTION COEFF. K ==0.30000
DEPTH = 1.50*****
BURIED POLE SOURCE, SURFACE BIPOLE RECEIVER
*****UPPER SOURCE= 0.000
LOWER SOURCE= 2,500
X-SOURCE= -2,000
Y-SOURCE= 0.000

Y-PROFILE= 0.000

X-RECEIVER APPARENT APPARENT
POSITION RESISTIVITY POLARIZABILITY

-10.500	.999472E+00	.845910E-03
-10.000	.999455E+00	.872770E-03
-9.500	.999437E+00	.901742E-03
-9.000	.999417E+00	.933225E-03
-8.500	.999395E+00	.967768E-03
-8.000	.999371E+00	.100616E-02
-7.500	.999344E+00	.104957E-02
-7.000	.999312E+00	.109979E-02
-6.500	.999275E+00	.115970E-02
-6.000	.999228E+00	.123408E-02
-5.500	.999167E+00	.133128E-02
-5.000	.999082E+00	.146668E-02
-4.500	.998955E+00	.167017E-02
-4.000	.998743E+00	.200904E-02
-3.500	.998354E+00	.263302E-02
-3.000	.997521E+00	.397423E-02
-2.500	.995003E+00	.805302E-02
-2.000	.170141E+39	.100000E+01
-1.500	.100026E+01	-.637920E-03
-1.000	.992922E+00	.111229E-01
-0.500	.985694E+00	.229200E-01
0.000	.984036E+00	.257835E-01
0.500	.991517E+00	.137235E-01
1.000	.100140E+01	-.207615E-02
1.500	.100668E+01	-.104166E-01
2.000	.100775E+01	-.121172E-01
2.500	.100710E+01	-.111190E-01
3.000	.100608E+01	-.954312E-02

3,500	,100514E+01	-,806429E-02
4,000	,100434E+01	-,681831E-02
4,500	,100371E+01	-,582415E-02
5,000	,100320E+01	-,503712E-02
5,500	,100280E+01	-,440573E-02
6,000	,100248E+01	-,389492E-02
6,500	,100221E+01	-,347673E-02
7,000	,100199E+01	-,313029E-02
7,500	,100181E+01	-,284000E-02
8,000	,100165E+01	-,259418E-02

SURFACE-TO-HOLE
SURFACE, FIXED POLE SOURCE
BURIED, MOVING BIPOLE RECEIVER

INPUT DATA

AR=4 SURFACE-TO-HOLEFIXED SURFACE SOURCE MOVING BURIED RECEIVER
4,
-0,3,2,5
2,,0,0,,5,10,
-2,,0,,0,
.5,,5,,5

OUTPUT DATA

3-DIMENSIONAL IP AND RESISTIVITY MODELLING PROGRAM

DATE: 24-JUN-76 TIME: 11:36 PAGE:

INPUT CONTROL PARAMETERS:

KVALU -0.30000
DEPTH 2.50
EXECU

DESCRIPTION OF MODEL: AR=4 SURFACE-TO-HOLE
FIXED SURFACE SOURCE
MOVING BURIED
RECEIVER

Z-COORDINATE (X,Y)-COORDINATES IN PAIRS -----

TOP	-0.500000	0.000000	0.000000	0.1443376	0.4082483
CONTOUR 1	-0.250000	0.2886751	0.3227486	-0.1443376	0.4082483
		0.0000000	0.4330127	-0.2886751	-0.3227486
-0.2886751	0.3227486	-0.4330127	0.0000000	-0.1443376	0.4082483
-0.1443376	-0.4082483	0.0000000	-0.4330127	0.1443376	-0.4082483
0.2886751	-0.3227486	0.4330127	0.0000000		
CONTOUR 2	0.0000000	0.1666667	0.4714045	0.0000000	0.5000000
		-0.1666667	0.4714045	-0.3333333	0.3726760
-0.5000000	0.0000000	-0.3333333	-0.3726780	-0.1666667	-0.4714045
0.0000000	-0.5000000	0.1666667	-0.4714045	0.3333333	-0.3726780
0.5000000	0.0000000	0.3333333	0.3726780		
CONTOUR 3	0.2500000	0.0000000	0.4330127	-0.1443376	0.4082483
		-0.2886751	0.3227486	-0.4330127	0.0000000
-0.2886751	-0.3227486	-0.1443376	-0.4082483	0.0000000	-0.4330127
0.1443376	-0.4082483	0.2886751	-0.3227486	0.4330127	0.0000000
0.2886751	0.3227486	0.1443376	0.4082483		
BOTTOM	0.5000000	0.0000000	0.0000000	0.0000000	

3-DIMENSIONAL IP AND RESISTIVITY MODELLING PROGRAM

DATE: 24-JUN-76

TIME: 11:36

DESCRIPTION OF MODEL: AR=4 SURFACE-TO-HOLE
FIXED SURFACE SOURCE
MOVING BURIED
RECEIVER

REFLECTION COEFF. K = -0.30000

DEPTH = 2.50

SURFACE POLE SOURCE, BURIED BIPOLE RECEIVER

X-SOURCE= -2.000

Y-SOURCE= 0.000

X-RECEIVER= 2.000

Y-RECEIVER= 0.000

RECEIVER DEPTH	APPARENT RESISTIVITY	APPARENT POLARIZABILITY
----------------	----------------------	-------------------------

1.000	.983445E+00	.265014E-01
1.500	.980005E+00	.320888E-01
2.000	.979539E+00	.328147E-01
2.500	.985392E+00	.232380E-01
3.000	.995893E+00	.639179E-02
3.500	.100507E+01	-.800211E-02
4.000	.100950E+01	-.148110E-01
4.500	.101014E+01	-.157482E-01
5.000	.100906E+01	-.140628E-01
5.500	.100760E+01	-.117716E-01
6.000	.100622E+01	-.962915E-02
6.500	.100506E+01	-.783020E-02
7.000	.100416E+01	-.642914E-02
7.500	.100347E+01	-.534990E-02
8.000	.100292E+01	-.448972E-02
8.500	.100248E+01	-.380896E-02
9.000	.100213E+01	-.326444E-02
9.500	.100185E+01	-.282392E-02

Appendix B
Development of theoretical expressions

Using image theory and applying the following boundary conditions:

- (1) there is no vertical current flow at the air-earth interface,
- (2) the potential must be continuous across regions of different conductivity,
- (3) the normal component of current flow must be continuous across regions of different conductivity,
- and (4) in the vicinity of the current electrode the expression for the potential must converge to the expression for a point source in a homogenous half-space.

The expression for the potential, U_M , given by Barnett (1972) can be modified for a buried point, M_d , due to a buried point source at A_d and can be written as

$$U_{M_d} = \frac{\rho_1 I}{4\pi} \left(\frac{1}{R_{A_d M_d}} + \frac{1}{R_{A_d^* M_d}} \right) + \left\{ \int_C \sigma_p \frac{1}{R_{pM_d}} dC + \int_{\tilde{C}} \sigma_p^* \frac{1}{R_{pM_d}} d\tilde{C} \right\} \quad (B1)$$

where σ_p and σ_p^* represent the fictitious surface charge density distributions for the body, C , and its image, \tilde{C} , and $R_{A_d M_d}$, $R_{A_d^* M_d}$, R_{pM_d} , and $R_{pM_d^*}$ are the distances shown in fig. 2. The problem of solving equation (7) is to solve for the fictitious charge densities and then to perform the necessary integrations over C and \tilde{C} .

The expression for σ_Q at a point Q on C is solved by applying boundary conditions (2) and (3). Boundary condition (2) can be satisfied by considering the potential at points Q_1 and Q_2 on opposite

sides of the boundary. Then for a wholespace, the potential at points Q_1 and Q_2 can be expressed as

$$U_{Q_1} = \frac{\rho_1 I}{4\pi R_{A_d Q_1}} + \frac{1}{4\pi} \int_C \sigma_p \left(\frac{1}{R_{PQ_1}} \right) dC , \quad (B2)$$

and

$$U_{Q_2} = \frac{\rho_1 I}{4\pi R_{A_d Q_2}} + \frac{1}{4\pi} \int_C \sigma_p \left(\frac{1}{R_{PQ_2}} \right) dC , \quad (B3)$$

As Q_1 and Q_2 approach one another, $U_{Q_1}|_Q = U_{Q_2}|_Q$, and the second

boundary condition is satisfied. Applying the third boundary condition,

$\frac{1}{\rho_1} \frac{\partial U_{Q_1}}{\partial v}|_C = \frac{1}{\rho_2} \frac{\partial U_{Q_2}}{\partial v}|_C$, we obtain the expression

$$\begin{aligned} & \left[\frac{1}{\rho_1} \frac{\partial}{\partial v} \left[\frac{\rho_1 I}{4\pi R_{A_d Q_1}} + \frac{1}{4\pi} \int_C \sigma_p \left(\frac{1}{R_{PQ_1}} \right) dC \right] \right]_{Q_1 \rightarrow Q} = \\ & \left[\frac{1}{\rho_2} \frac{\partial}{\partial v} \left[\frac{\rho_1 I}{4\pi R_{A_d Q_2}} + \frac{1}{4\pi} \int_C \sigma_p \left(\frac{1}{R_{PQ_2}} \right) dC \right] \right]_{Q_2 \rightarrow Q} \end{aligned} \quad (B4)$$

It is not obvious that the expressions on the left and right hand sides of the equal sign are equal. In fact, the integrals are improper when $p \rightarrow Q$. It can be shown (Barnett, 1972, p. 54) that these integrals can be solved so that

$$\lim_{\substack{Q_1 \rightarrow Q \\ p=q}} \left\{ \frac{1}{4\pi} \int_C \sigma_p \frac{\partial}{\partial v} \left(\frac{1}{R_{PQ_1}} \right) dC \right\} = -\frac{1}{2} \sigma_Q \quad (B5)$$

and

$$\lim_{\substack{Q_2 \rightarrow Q \\ p=q}} \left\{ \frac{1}{4\pi} \int_C \sigma_p \frac{\partial}{\partial v} \left(\frac{1}{R_{PQ_1}} \right) dC \right\} = + \frac{1}{2} \sigma_Q, \quad (B6)$$

Utilizing this singularity, the boundary condition in the limit can be expressed as

$$\sigma_Q = \frac{K\rho_1 I}{2\pi} \frac{\partial}{\partial v} \left(\frac{1}{R_{A_d Q}} \right) + \frac{K}{2\pi} \int_{C'} \sigma_p \frac{\partial}{\partial v} \left(\frac{1}{R_{PQ}} \right) dC \quad (B7)$$

where $K = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}$, and C' is the whole surface of the body excluding that element at point Q.

Applying image theory, the expressions for the halfspace can be written as:

$$\sigma_Q = \frac{K\rho_1 I}{2\pi} \left\{ \frac{\partial}{\partial v} \left(\frac{1}{R_{A_d Q}} \right) + \frac{\partial}{\partial v} \left(\frac{1}{R_{A_d^v Q}} \right) \right\} + \quad (B8)$$

$$\frac{K}{2\pi} \left\{ \int_{C'} \sigma_p \frac{\partial}{\partial v} \left(\frac{1}{R_{PQ}} \right) dC + \int_{\tilde{C}} \sigma_p \frac{\partial}{\partial v} \left(\frac{1}{R_{PQ}^v} \right) d\tilde{C} \right\}$$

and

$$U_{M_d} = \frac{\rho_1 I}{4\pi} \left\{ \frac{1}{R_{A_d M_d}} + \frac{1}{R_{A_d^v M_d}} \right\} + \frac{1}{4\pi} \left\{ \int_C \sigma_p \left(\frac{i}{R_{PM_d}} \right) dC + \int_{\tilde{C}} \sigma_p \left(\frac{1}{R_{PM_d}^v} \right) d\tilde{C} \right\} \quad (B9)$$

assuming that $\sigma_p^v = \sigma_p$.

The problem at this point is to solve the integral expressions in equations (B8) and (B9) for the charge density distribution, σ_p . The charge density can be expressed as $S_p = \frac{\sigma_p}{\rho_1 I}$, so that equations (B8) and (B9) become

$$\frac{2\pi}{K} S_Q = \left\{ \frac{\partial}{\partial v} \left(\frac{1}{R_{A_d Q}} \right) + \frac{\partial}{\partial v} \left(\frac{i}{R_{A_d Q}} \right) \right\} + \quad (B10)$$

$$\int_C S_p \frac{\partial}{\partial v} \left(\frac{1}{R_{PQ}} \right) dC + \int_{\tilde{C}} S_p \frac{\partial}{\partial v} \left(\frac{1}{R_{PQ}} \right) d\tilde{C}$$

and

$$U_M = \frac{\rho_1 I}{4\pi} \left\{ \frac{1}{R_{A_d M_d}} + \frac{1}{R_{A_d M_d}} + \int_C S_p \left(\frac{1}{R_{P M_d}} \right) dC + \int_{\tilde{C}} S_p \left(\frac{1}{R_{P M_d}} \right) d\tilde{C} \right\} \quad (B11)$$

The unknown function S_p is approximated by a set of N discrete functions, so that

$$S_p \approx \sum_{j=1}^N S_j \alpha_j. \quad (B12)$$

The functions S_j are called the "expansion functions" or "basis functions" while α_j are constants to be determined (Harrington, 1968).

In order to put equation (B10) in terms of point p rather than point Q , the singularity condition (equations (B4), (B5), and (B6)) is applied to equation (B10) and the basis functions expressed in

equation (B12), so that

$$\begin{aligned} \frac{2\pi}{K} S_i b_{ii} &= \frac{\partial}{\partial v} \left(\frac{1}{R_{A_d Q}} \right) + \frac{\partial}{\partial v} \left(\frac{1}{R_{A_d Q}^*} \right) + \\ \sum_{\substack{j=1 \\ j \neq i}}^N S_j \int_C \alpha_j \frac{\partial}{\partial v} \left(\frac{1}{R_{PQ}} \right) dC + \sum_{j=1}^N S_j \int_{\tilde{C}} \alpha_j \frac{\partial}{\partial v} \left(\frac{1}{R_{PQ}^*} \right) d\tilde{C} \end{aligned} \quad (B13)$$

and

$$U_{M_d} = \frac{\rho_1 I}{4\pi} \left\{ \frac{1}{R_{A_d M_d}} + \frac{1}{R_{A_d M_d}^*} + \sum_{j=1}^N S_j \int_{\tilde{C}} \alpha_j \left(\frac{1}{R_{P_d M_d}^*} \right) d\tilde{C} + \right. \\ \left. \sum_{j=1}^N S_j \int_C \alpha_j \left(\frac{1}{R_{P_d M_d}} \right) dC \right\} \quad (B14)$$

where

$$b_{ii} = -\frac{1}{4\pi} \int_C \alpha_j \frac{\partial}{\partial v} \left(\frac{1}{R_{PQ_1}} \right) dC - \int_C \alpha_j \frac{\partial}{\partial v} \left(\frac{1}{R_{PQ_2}} \right) dC \quad (B15)$$

as $P \rightarrow Q$ and $Q_1 \rightarrow Q$ and $Q_2 \rightarrow Q$.

The surface C can be divided into triangular subareas over which the source density is assumed to be constant. The constants for the basis functions can be expressed as follows:

$$\begin{aligned} \alpha_j &= 1 \text{ over subarea } C_j, \\ &= 0 \text{ over } C_i, \text{ where } i \neq j. \end{aligned}$$

Using this approximation and equations (B4), (B5), (B6), (B12), and (B15) we find that $b_{ii} = 1$. Substituting this into equation (B12), equations (B13) and (B14) can be put in matrix form by letting

$$F_i = \frac{\partial}{\partial v} \left(\frac{1}{R_{A_d^Q}} \right) + \frac{\partial}{\partial v} \left(\frac{1}{R_{PQ}} \right), \quad (B16)$$

$$GB_{ij} = - \int_{C^i} \alpha_j \frac{\partial}{\partial v} \left(\frac{1}{R_{PQ}} \right) dC, \quad (B17)$$

$$GB_{ii} = \frac{2\pi}{K} b_{ii} \quad (B18)$$

$$GI_{ij} = - \int_{\tilde{C}} \alpha_j \frac{\partial}{\partial v} \left(\frac{1}{R_{PQ}} \right) d\tilde{C}, \quad (B19)$$

$$G_{ij} = GB_{ij} + GI_{ij} \quad (B20)$$

$$\text{and } H_i = \int_{\tilde{C}} \alpha_i \left(\frac{1}{R_{PQ}} \right) d\tilde{C} + \int_C \alpha_i \left(\frac{1}{R_{PQ}} \right) dC, \quad (B21)$$

Using these equations, the matrix form for equations (B13) and (B14) becomes:

$$F_i = \sum_{j=1}^N G_{ij} S_j \quad (B22)$$

and

$$U_{M_d} = \frac{\rho_1 I}{4\pi} \left\{ \frac{1}{R_{A_d^M}} + \frac{1}{R_{A_d^M}} + \sum_{i=1}^N S_i H_i \right\}. \quad (B23)$$

In order to calculate the apparent polarizability the equation,

$$\frac{\partial U_{M_d}}{\partial K} = \frac{\rho_1 I}{4\pi} \sum_{i=1}^N \frac{\partial S_i}{\partial K} H_i , \quad (B24)$$

needs to be calculated, Defining

$$T_i \triangleq \frac{\partial S_i}{\partial K} \text{ and } D_i \triangleq \frac{G B_{ii} S_i}{K} ,$$

we can write

$$\sum_{j=1}^N G_{ij} T_j = D_i \quad (B25)$$

and

$$\frac{\partial U_{M_d}}{\partial K} = \frac{\rho_1 I}{\pi} \sum_{i=1}^N T_i H_i \quad (B26)$$

The problem of mathematically modeling the resistivity and IP response at a buried receiver pair (M_d and N_d) due to a buried current source (A_d and B_d) in the presence of a three-dimensional body (C) reduces to the problem of solving equations (B22), (B23), (B25), and (B26) for the proper source-receiver combinations.

Appendix C

Program listing

C--TP3DDH-- INDUCED POLARIZATION (3-DIMENSIONAL) -- 7/23/74.

C

PROGRAM: IP3DDH

THIS PROGRAM COMPUTES DOWNHOLE AND SURFACE PROFILES OF NORMALIZED APPARENT RESISTIVITY (ρ_a / ρ_i) AND NORMALIZED APPARENT POLARIZABILITY ($(\eta_a - \eta_i) / (\eta_2 - \eta_i)$) ACROSS AN ARBITRARILY SHAPED THREE-DIMENSIONAL BODY, OF RESISTIVITY ρ_2 AND POLARIZABILITY η_2 , SET IN AN OTHERWISE HOMOGENEOUS HALF-SPACE OF RESISTIVITY ρ_i AND POLARIZABILITY η_i . THREE-DIMENSIONAL (POINT-SOURCE) DOWNHOLE ELECTRODES ARE USED.

THE PROGRAM IS WRITTEN IN FORTRAN-IV, AND WAS DEVELOPED ON A DIGITAL EQUIPMENT CORPORATION MODEL PDP-10 COMPUTER.

C AN ELLIPSOID OF REVOLUTION (WITH 72 FACETS) IS GENERATED IN
C SUBROUTINE BODY3D. ALL THAT IS NEEDED TO GENERATE THIS BODY
C ARE THE 'A', 'B', AND 'C' (X,Y,&Z) HALF WIDTHS OF THE BODY
C (SEE SUBROUTINE BODY3D)

RESULTS ARE OUTPUT ON DISK

THE FIRST PAGE OF OUTPUT. PROVIDES A RECORD OF THE INPUT DATA.
SUBSEQUENT PAGES GIVE APPARENT RESISTIVITY AND
APPARENT POLARIZABILITY VALUES.

PROGRAM IP300H DOES NOT CONTAIN A PROVISION FOR MULTIPLE BODIES

PASANETER DESCRIPTION: C

XPD AND YVAL ARE THE (X AND Y) RECEIVER POSITIONS
XPDOWN, YDOWN, ZDOWN(XD, YD, ZD) ARE THE SOURCE CO-ORDINATES

TL=TOTAL LENGTH OF SURFACE PROFILE (PROFILE STARTS AT -11.
SIPOLE UNITS FROM THE BODY CENTER)

HD=HOLE DEPTH (IN RECEIVER BIPOLE UNITS)

PS = SPACING BETWEEN MEASUREMENT POINTS (MUST BE AN INTEGER
DIVISER OF 1, LESS THAN OR EQUAL TO 1)

(TL OR HD)/PS MUST BE LESS THAN 50. IF MORE THAN 50 DATA POINTS ARE DESIRED, THE DIMENSIONS CAN BE INCREASED

NSPA=N-SPACING FOR SINGLE HALF BIPOLE-BIPOLE CONFIGURATION
FOR N=1 THE SPACING BETWEEN THE B SOURCE ELECTRODE AND THE M
RECEIVER ELECTRODE IS 1, FOR N=2 THE SPACING IS 2, ETC.

C*****ARRAY CONFIGURATIONS*****
C AR=1.,BURIED BIPOLE SOURCE,SURFACE BIPOLE RECEIVER
C AR=2.,BURIED POLE SOURCE, SURFACE RIPPLE RECEIVER
C AR=3.,BURIED POLE SOURCE, BURIED BIPOLE RECEIVER
C AR=4.,SURFACE POLE FIXED SOURCE,BURIED RIPPLE RECEIVER
C AR=5.,BURIED BIPOLE FIXED SOURCE, BURIED BIPOLE RECEIVER
C AR=6.,BURIED BIPOLE MOVING SOURCE, BURIED BIPOLE RECEIVER
C AR=7.,SINGLE HOLE CONFIGURATION

ALL READ STATEMENTS ARE IN THE MAIN PROGRAM AND SUBROUTINE BODY3D

.....
REQUIRED SUBROUTINES: PEAD3D, IPAG3D,
DIRC3D, GB3D, GI3D, H3D, F3D,
GAUS10, FX1, FX2,
DECOMP, SOLVE,
NPAG3D, ARAY3D, OUTPUT, BODY3D.

DEVICE SPECIFICATIONS:

13 = IN1 INPUT = CONTROL CARDS WITH PROGRAM PARAMETERS.
13 = IN2 INPUT = DESCRIPTION OF MODEL, FOLLOWED BY COORDINATES
OF THE VERTICES.
14 = IOUT1,IOUT2,IOUT3

NOTE: FOR HOLE-TO-SURFACE, SURFACE-TO-HOLE, AND HOLE-TO-HOLE ARRAYS
OUTPUT POINTS ARE AT THE MIDPOINT OF THE RECEIVER ELECTRODES. FOR
SINGLE HOLE ARRAY OUTPUT POINTS ARE AT THE MIDPOINT BETWEEN THE
SOURCE AND RECEIVER ELECTRODES((B-M)/2)

-----C.T.BARNETT-----APRIL 1972-----

C*****MODIFIED FOR BURIED ELECTRODE ARRAYS*****
C JEFF DANIELS
C U.S.GEOLOGICAL SURVEY
C MAY 1976

LOGICAL CHECK

COMMON /BLOK2/ XM(75),YM(75),ZM(75)
COMMON /BLOK7/ F(75)
COMMON /BLOK13/ GB(75,75)
COMMON /BLOK14/ HH(55,75),SS(75),TT(75),S(75),T(75)
COMMON /BLOK15/ G(75,75)
COMMON /BLOK17/ H(75)
COMMON /ISPECS/ IN1,IN2,IOUT1,IOUT2,IOUT3
COMMON /PARAM / IARRAY(7),ARVAL(7),DEPTH(7),ANGLE(8),YVAL(8),
& NARR,NKV,NDPTH,NDIP,NYV,IPLOT
COMMON /DATIME/ LABEL(16),JDATE(2),ITIME,NPAGES,IPAGE
COMMON /POLY / NFACES
COMMON /RESULT/ APRES(50),APIP(50),AR,PS,IR2,MSPA,ZPI

```

DIMENSION D(75),GBDIAG(75),GIDIAG(75),
&XDOWN(8),YDOWN(8),ZDOWN(8),H1(75)
DATA IN1,IN2,IOUT1,IOUT2,IOUT3 /2*13,3*14/
C
  IPLOT=1
C*****INPUT THE CONTROL PARAMETERS
C INPUT THE CONTROL PARAMETERS
C*****
READ(IN2,11)LABEL
READ(IN1,10)AR
14 FORMAT(3I)
  READ(IN1,10) AKVAL(1),DEPTH(1)
  IF(AR.EQ.1.) GO TO 71
  IF(AR.EQ.2.) GO TO 72
  IF(AR.EQ.3.) GO TO 73
  IF(AR.EQ.4.) GO TO 74
  IF(AR.EQ.5.) GO TO 75
  IF(AR.EQ.6.) GO TO 76
  IF(AR.EQ.7.) GO TO 77
-- 71 READ(IN1,10) YVAL(1),PS,TL
  READ(IN1,10) XDOWN(1),YDOWN(1),ZDOWN(1),ZDOWN(2)
  IDH=0
  GO TO 78
  72 READ(IN1,10) YVAL(1),PS,TL
  READ(IN1,10) XDOWN(1),YDOWN(1),ZDOWN(1)
  IDH=0
  GO TO 78
  73 READ(IN1,10) XPD,YVAL(1),PS,HD
  READ(IN1,10) XDOWN(1),YDOWN(1),ZDOWN(1)
  IDH=1
  GO TO 78
  74 READ(IN1,10) XPD,YVAL(1),PS,HD
  READ(IN1,10) XDOWN(1),YDOWN(1),ZDOWN(1)
  IDH=1
  GO TO 78
  75 READ(IN1,10) XPD,YVAL(1),PS,HD
  READ(IN1,10) XDOWN(1),YDOWN(1),ZDOWN(1),ZDOWN(2)
  IDH=1
  GO TO 78
  76 READ(IN1,10) XPD,YVAL(1),PS,HD
  READ(IN1,10) XDOWN(1),YDOWN(1)
  IDH=1
  GO TO 78
  77 READ(IN1,10) XPD,YVAL(1),PS,HD
  READ(IN1,14)NSPA
  XDOWN(1)=XPD
  YDOWN(1)=YVAL(1)
  IDH=1
  78 CONTINUE
10 FORMAT(7F)
C
C      ---- INPUT CONTROL PARAMETERS
C      NPAGES=1+NARR*NKV*NDPTH*NDIP*NYV
C
C      ---- INPUT DESCRIPTION OF MODEL
11 FORMAT (16A5)
C
C      ---- INPUT COORDINATES OF APICES OF POLYHEDRON
C      ---- (INCREASE PROGRAM DIMENSION STATEMENTS IF NECESSARY)
CALL READ30

```

```

C
C      ---- OBTAIN TIME & DATE, THEN PRINT OUT PAGE 1
C      ---- (RECORD OF INPUT DATA)
C      CALL DATE (IDATE)
C      CALL TIME (ITIME)
C      CALL IPAG3D
C
C      ---- DETERMINE OUTWARD-DIRECTED NORMALS AND
C      ---- TRANSFORM TO UVW-COORDINATES
C      CALL DIRC3D
C
C      ---- COMPUTE BODY-BODY INTERACTION TERMS
C      CALL GB3D
C      DO 80 I=1,NFACES
C        80 GBDIAG(I)=GB(I,I)
C
C      ---- ROTATE BODY TO REQUIRED DIP ANGLES
C      ---- RECOMPUTE OUTWARD-DIRECTED NORMALS
C      CALL DIRC3D
C      90 CONTINUE
C
C      ---- ADJUST BODY TO REQUIRED DEPTHS
C      Z0=DEPTH(1)
C
C      ---- COMPUTE BODY-IMAGE INTERACTION TERMS
C      ---- ASSEMBLE OFF-DIAGONAL MATRIX ELEMENTS
C      CALL GI3D (Z0)
C      DO 100 I=1,NFACES
C        GIDIAG(I)=G(I,I)
C        DO 100 J=1,NFACES
C          IF (J.EQ.I) GO TO 100
C          G(I,J)=GS(I,J)+G(I,J)
C      2068 FORMAT(2X,'ROW # ',IS)
C      100 CONTINUE
C
C      ---- ADJUST Y-COORDINATE FOR REQUIRED PROFILES
C      CHECK=.FALSE.
C      DO 1000 IY=1,NYV
C        Y0=YVAL(IY)
C        YPD=Y0
C
C      ---- COMPUTE RECEIVER FUNCTIONS AT VARIOUS
C      ---- POSITIONS ALONG THE TRAVERSE
C      ****FOR HOLE TO SURFACE ARRAYS ONLY****
C      IF(AR.GE.3.) GO TO 1003
C      1002 CONTINUE
C        IR1=1
C        IR2=IFIX(TL/PS)
C        DO 200 IX=IR1,IR2+1
C          XP=-11.5+ FLOAT(IX)*PS
C          CALL H3D (XP,Y0,Z0,IDH)
C      6020 FORMAT(2X,IS,3E12.6)
C        DO 200 I=1,NFACES
C          200 HH(IX,I)=H(I)
C
C        GO TO 1001
C      ***COMPUTE RECEIVER FUNCTIONS AT VARIOUS POINTS (AT PS SPACING)
C      ALONG THE RECEIVER BOREHOLE,(FOR HOLE-TO-HOLE ARRAYS ONLY)
C      1003 CONTINUE
C        IR1=1

```

```

IR2=IFIX(HD/PS)
DO 201 IX=IR1,IR2
ZP=PS*FLOAT(IX)
ZPP=ZO+ZP
CALL H3D(XPD,YPD,ZPP,IDH)
DO 202 I5=1,NFACES
202 H1(I5)=H(I5)
ZPP=ZO-ZP
CALL H3D(XPD,YPD,ZPP,IDH)
DO 201 I=1,NFACES
201 HH(IX,I)=H(I)+H1(I)
1001 CONTINUE
C     ---- ADJUST TO REQUIRED REFLECTION COEFFICIENTS
DO 1000 I4=1,NKV
IF (CHECK.AND.NKV.EQ.1) GO TO 400
AK=AKVAL(I4)
BK=0.5*(1.0-AK*AK)
AKI=1.0/AK
AKISQ=AKI*AKI
C     ---- ASSEMBLE DIAGONAL MATRIX ELEMENTS
DO 300 I=1,NFACES
300 G(I,I)=AKI*GBDIAG(I)+GIDIAG(I)
C     ---- DECOMPOSE MATRIX INTO UPPER AND LOWER
C     ---- TRIANGULAR FACTORS
CALL DECOMP
C     CHECK=.TRUE.
400 CONTINUE
C
C ****
C     ADJUST COORDINATES OF DOWNHOLE ELECTRODE
C ****
ISPREV=0
ZPT=0.0
XD=XDOWN(1)
YD=YDOWN(1)
IF(AR.GE.2.0.AND.AR.LE.4.0) GO TO 82
IF(AR.EQ.6.0) GO TO 33
IF(AR.EQ.7.0) GO TO 84
81 IS2=1
ZDA=ZDOWN(1)
ZDB=ZDOWN(2)
XDDD=XD
GO TO 88
82 IS2=1
ZDA=0.0
ZDB=ZDOWN(1)
XDDD=1.E+08
GO TO 88
83 IS2=IR2
ZDA=0.0
GO TO 88
84 IS2=IR2-IFIX(1./PS+NSPA/PS)
88 CONTINUE
CALL NPAG3D(DIP,ZD,YD,AK,IPLOT,INDEX)
DO 1000 I6=1,IS2
IF(AR.GE.2.0.AND.AR.LE.4.0) GO TO 96

```

```

    IF(AR.GE.6.) GO TO 93
91  IF(ZDA)96,96,966
93  ZDA=PS*FLOAT(I6)
     ZDB=ZDA+1.0
     GO TO 966
*****
C COMPUTE S AND T FUNCTIONS FOR SURFACE ELECTRODE
96  CALL F3D(X000,Y0,Z0,0,ZDA)
    CALL SOLVE(F,S)
    DO 969 I=1,NFACES
969 D(I)=S(I)*AKISQ*GBOIAG(I)
    CALL SOLVE(D,I)
    DO 962 I=1,NFACES
     SS(I)=S(I)
962 TT(I)=T(I)
    IF(AP.NE.4) GO TO 963
    ZDB=ZDA
    CALL F3D(X0,Y0,Z0,0,ZDB)
    CALL SOLVE(F,S)
    DO 959 I=1,NFACES
959 D(I)=S(I)*AKISQ*GBOIAG(I)
    CALL SOLVE(D,T)
    GO TO 945
*****
C COMPUTE S AND T FUNCTIONS FOR DOWNHOLE ELECTRODES
966 CALL F3D(X0,Y0,Z0,1,ZDA)
    CALL SOLVE(F,S)
    DO 971 I=1,NFACES
971 D(I)=S(I)*AKISQ*GBOIAG(I)
    CALL SOLVE(D,T)
    DO 967 I=1,NFACES
     SS(I)=S(I)
967 TT(I)=T(I)
963 CALL F3D(X0,Y0,Z0,1,ZDB)
    CALL SOLVE(F,S)
    DO 964 I=1,NFACES
5002 FORMAT(3(2X,E12.6))
5001 FORMAT(10X,I5,2E12.6)
964 D(I)=S(I)*AKISQ*GBOIAG(I)
    CALL SOLVE(D,T)
945 CONTINUE
C
C      ---- COMPUTE AND OUTPUT PROFILES FOR THE VARIOUS
C      ---- ARRAYS
C      IPAGE=IPAGE+1
621 FORMAT(4(2X,E12.6))
I66=I6
CALL ARAY3D(AK,BK,1DH,XPD,YPD,X0,Y0,ZD,ZDA,ZDB,I66)
ZPT=1.0
1000 CONTINUE
CALL OUTPUT (1DH,IPLOT,XPD,YPD,X0,Y0,ZD,ZDA,ZDB)
C
C      STOP
C
C
C
C      SUBROUTINE READ3D
C
```

C THIS SUBROUTINE CALLS BODY3D TO CALCULATE THE COORDINATES OF THE
C APICES OF THE POLYHEDRON REPRESENTING THE ARBITRARILY SHAPED BODY.
C IT THEN ASSEMBLES THE INDIVIDUAL TRIANGULAR FACETS IN A LOGICAL
C ORDER, SO THAT THE OUTWARD-DIRECTED NORMALS TO EACH FACE CAN BE
C DETERMINED (SEE SUBROUTINE DIRC3D).

C THE POLYHEDRON MUST HAVE A TOP AND A BOTTOM APEX. OTHER APICES
C MUST BE EVENLY DISTRIBUTED AROUND CONTOURS OF CONSTANT Z.
C EACH CONTOUR MUST HAVE THE SAME NUMBER OF APICES. OFFSET APICES
C ON ADJACENT CONTOURS TO OBTAIN MORE EQUILATERAL FACETS.

C INPUT

C NC = NUMBER OF CONTOURS,
C NV = NUMBER OF APICES PER CONTOUR,
C ZT,VZ,ZB = Z-COORDINATES OF TOP, SUCCESSIVE CONTOURS, AND
C BOTTOM.
C XT,YT = XY-COORDINATES OF TOP APEX.
C VX,VY = XY-COORDINATES OF CONTOUR APICES. INPUT THESE
C CLOCKWISE (PLAN-VIEW) AROUND EACH CONTOUR IN
C DESCENDING ORDER, STARTING ONE POINT FURTHER
C CLOCKWISE ON SUCCESSIVE CONTOURS.
C XB,YB = XY-COORDINATES OF BOTTOM APEX.

C -----C.T.BARNETT-----APRIL 1972-----

C*****MODIFIED BY JEFF DANIELS ::::MAY 1976 *****

C
COMMON /BLOK1/ XF(75,3),YF(75,3),ZF(75,3)
COMMON /BLOK8 / VX(12,12),VY(12,12),VZ(12)
COMMON /BLOK9 / XT,YT,ZT,XB,YB,ZB
COMMON /BLOK12/ NC,NV
COMMON /ISPPCS/ IN1,IN2,IOUT1,IOUT2,IOUT3
COMMON /POLY / NFACES

C
COMMON/BODY/VXX(100),VYY(100),VZZ(5)
CALL BODY3D
NV2=2*NV
NFACES=NC*NV2
ZT=VZZ(1)
DO 1022 I=2,(NC+1)
1022 VZ(I-1)=VZZ(I)
ZB=VZZ(NC+2)
IST=1
XT=VXX(1)
YT=VYY(1)
DO 100 J=1,NC
DO 100 J=1,NV
IST=IST+1
VX(I,J)=VXX(IST)
100 VY(I,J)=VYY(IST)
IST=IST+1
XB=VXX(IST)
YB=VYY(IST)

C
KT1=NFACES-NV
DO 101 J=1,NV
KT1=KT1+1
J1=J+1
IF (J1.GT.NV) J1=1
XF(J,1)=VX(1,J)

```

    YF(J,1)=VY(1,J)
    ZF(J,1)=VZ(1)
    XF(J,2)=VX(1,J1)
    YF(J,2)=VY(1,J1)
    ZF(J,2)=VZ(1)
    XF(J,3)=XT
    YF(J,3)=YT
    ZF(J,3)=ZT
    XF(KT1,1)=VX(NC,J)
    YF(KT1,1)=VY(NC,J)
    ZF(KT1,1)=VZ(NC)
    XF(KT1,2)=XB
    YF(KT1,2)=YB
    ZF(KT1,2)=ZB
    XF(KT1,3)=VX(NC,J1)
    YF(KT1,3)=VY(NC,J1)
101 ZF(KT1,3)=VZ(NC)

```

```

C
    IF (NC.EQ.1) RETURN
    ND=NC-1
C

```

```

DO 102 I=1,ND
I1=I+1
KT2=I*NV2
KT1=KT2-NV
DO 102 J=1,NV
J1=J+1
IF (J1.GT.NV) J1=1
KT1=KT1+1
XF(KT1,1)=VX(I,J)
YF(KT1,1)=VY(I,J)
ZF(KT1,1)=VZ(I)
XF(KT1,2)=VX(I1,J)
YF(KT1,2)=VY(I1,J)
ZF(KT1,2)=VZ(I1)
XF(KT1,3)=VX(I,J1)
YF(KT1,3)=VY(I,J1)
ZF(KT1,3)=VZ(I)

```

```

C
    KT2=KT2+1
    XF(KT2,1)=VX(I,J1)
    YF(KT2,1)=VY(I,J1)
    ZF(KT2,1)=VZ(I)
    XF(KT2,2)=VX(I1,J)
    YF(KT2,2)=VY(I1,J)
    ZF(KT2,2)=VZ(I1)
    XF(KT2,3)=VX(I1,J1)
    YF(KT2,3)=VY(I1,J1)
102 ZF(KT2,3)=VZ(I1)

```

```

C
    RETURN
    END
C
C
C
C

```

SUBROUTINE IP1G3D

THIS SUBROUTINE PRINTS OUT PAGE 1 OF THE OUTPUT FOR EACH MODEL.
THIS GIVES THE DATE AND TIME OF EXECUTION, THE INPUT CONTROL

PARAMETERS, AND A DESCRIPTION OF THE BODY FOLLOWED BY A PRINT-OUT
OF THE BODY COORDINATES.

-----C.T.BARNETT-----APRIL 1972-----

```
COMMON /BLOK8 / VX(12,12),VY(12,12),VZ(12)
COMMON /BLOK9 / XT,YT,ZT,XB,YB,ZB
COMMON /BLOK12/ NC,NV
COMMON /DATE/I DATE(16),IDATE(2),ITIME,NPAGES,IPAGE
COMMON /SPEC/ ID1, ID2,IOUT1,IOUT2, IDUT3
COMMON /PARAM / IARRAY(7),AKVAL(7),DEPTH(7),ANGLE(8),YVAL(8),
& NARR,NKV,NDPTH,NOIP,NYV,IPLOT
COMMON /POLY / NFACES

C
      WRITE (IOUT2,12) IDATE,ITIME
      WRITE (IOUT2,13)
      WRITE (IOUT2,15) (AKVAL(I),I=1,NKV)
      WRITE (IOUT2,16) (DEPTH(I),I=1,NDPTH)
      WRITE (IOUT2,20)
      WRITE (IOUT2,21) LABEL
      WRITE (IOUT2,22)
      WRITE (IOUT2,23) ZT,XT,YT
      DO 100 I=1,NC
      WRITE (IOUT2,24) (I,VZ(I),(VX(I,J),VY(I,J),J=1,2))
      WRITE (IOUT2,25)(VX(I,J),VY(I,J),J=3,4)
      WRITE (IOUT2,32)(VX(I,J),VY(I,J),J=5,7)
      WRITE (IOUT2,32)(VX(I,J),VY(I,J),J=8,10)
100   WRITE (IOUT2,32)(VX(I,J),VY(I,J),J=11,12)
      WRITE (IOUT2,26) ZB,XB,YB
      IF (IPLOT.EQ.1) RETURN
      WRITE (IOUT3,27) IDATE,ITIME
      WRITE (IOUT3,28) NPAGES,NFACES
      WRITE (IOUT3,29) LABEL
      WRITE (IOUT3,30) NC,NV
      WRITE (IOUT3,31) ZT,(VZ(I),I=1,NC),ZB
      WRITE (IOUT3,31) XT,YT
      DO 200 I=1,NC
200   WRITE (IOUT3,31) (VX(I,J),VY(I,J),J=1,NV)
      WRITE (IOUT3,31) XB,YB

C
      12 FORMAT (1H1,/,5X,'3-DIMENSIONAL IP AND RESISTIVITY MODELLING PROG
 & RAM',//5X,'DATE: ',2A5,10X,ITIME: !,A5,10X,'PAGE: !,I3,! OF!,I4)
      13 FORMAT (//,5X,'INPUT CONTROL PARAMETERS: !,/)
      15 FORMAT (5X,'KVALU! ,8X,7(F8.5,2X))
      16 FORMAT (5X,'DEPTH! ,5X,7(F8.2,2X))
      20 FORMAT (5X,'EXECU! ')
      21 FORMAT (///,5X,'DESCRIPTION OF MODEL: !,4X,4A5,/30X,4A5,/30X,4A5,
 &/30X,4A5)
      22 FORMAT (//,20X,'Z-COORDINATE!',5X,'(X,Y)-COORDINATES IN PAIRS  --
 &-----!//)
      23 FORMAT (5X,'TOP!',13X,F10.7,6X,F10.7,1X,F10.7)
      24 FORMAT (5X,'CONTOUR!',12,7X,F10.7,4X,2(2X,F10.7,1X,F10.7))
      25 FORMAT (35X,2(2X,F10.7,1X,F10.7))
      26 FORMAT (5X,'BOTTOM!',10X,F10.7,7X,F10.7,1X,F10.7)
      27 FORMAT (3A5)
      28 FORMAT (5X,2I5)
      29 FORMAT (16A5)
      30 FORMAT (2I10)
      31 FORMAT (8F10.7)
```



```
UU=1.0/SQRT(UX*UX+UY*UY+UZ*UZ)
```

```
C  
DNXI=WX*WW  
DNYI=WY*WW  
DNZI=WZ*WW  
DLXI=UX*UU  
DLYI=UY*UU  
DLZI=UZ*UU  
DMXI=DNYI*DLZI-DLYI*DNZI  
DMYI=DNZI*DLXI-DLZI*DNXI  
DMZI=DNXI*DLYI-DLXI*DNYI
```

```
C  
DLX(I)=DLXI  
DLY(I)=DLYI  
DLZ(I)=DLZI  
DMX(I)=DMXI  
DMY(I)=DMYI  
DMZ(I)=DMZI  
DNX(I)=DNXI  
DNY(I)=DNYI  
DNZ(I)=DNZI
```

```
C  
U1(I)=DLXI*X1+DLYI*Y1+DLZI*Z1  
U2(I)=DLXI*X2+DLYI*Y2+DLZI*Z2  
U3(I)=DLXI*X3+DLYI*Y3+DLZI*Z3  
V1(I)=DMXI*X1+DMYI*Y1+DNZI*Z1  
V2(I)=DMXI*X2+DMYI*Y2+DMZI*Z2  
W1(I)=DNXI*X1+DMYI*Y1+DNZI*Z1
```

```
C  
XM(I)=(X1+X2+X3)*0.33333333  
YM(I)=(Y1+Y2+Y3)*0.33333333  
ZM(I)=(Z1+Z2+Z3)*0.33333333
```

```
C 100 I=I+1  
C IF (I.LE.NFACES) GO TO 200  
C  
C RETURN  
C END
```

SUBROUTINE GB3D

```
C  
C GB3D COMPUTES THE INTERACTION TERMS BETWEEN RESPECTIVE FACES  
C OF THE BODY. THESE DEPEND PURELY ON THE BODY GEOMETRY AND ARE  
C INDEPENDENT OF THE BODY'S POSITION W.R.T. THE SURFACE OR THE  
C SOURCE.
```

```
C  
C PULSE-TYPE BASIS FUNCTIONS ARE USED.
```

```
C  
C AN APPROXIMATE VALUE IS FIRST CALCULATED (GBAPR). IF THE ABSOLUTE  
C VALUE OF THIS IS GREATER THAN 0.001, THEN THE EXACT VALUE IS  
C CALCULATED USING A 10-POINT GAUSS-LEGENDRE INTEGRATION FORMULA.
```

```
C  
C SUBROUTINE GAUS10 AND EXTERNAL FUNCTION FX1 ARE REQUIRED.
```

```
C  
C NFACES = NO. OF FACES
```

C -----C.T.BARNETT-----APRIL 1972-----C

```
COMMON /BLOK2 / XM(75),YM(75),ZM(75)
COMMON /BLOK3 / DLX(75),DLY(75),DLZ(75),DMX(75),
&DMY(75),DMZ(75)
COMMON /BLOK4 / DNX(75),DNY(75),DNZ(75)
COMMON /BLOK5 / U1(75),U2(75),U3(75),V1(75),V2(75),W1(75)
COMMON /BLOK13/ GE(75,75)
COMMON /BFX1 / P8,QB,RBR1,R1SQ,G1,G2,H1,H2
COMMON /POLY / NFACES
EXTERNAL FX1
DATA TWOPI /6.2831925/
```

C J=1

```
500 CONTINUE
DLXJ=DLX(J)
DLYJ=DLY(J)
DLZJ=DLZ(J)
DMXJ=DMX(J)
DMYJ=DMY(J)
DMZJ=DMZ(J)
DNXJ=DNX(J)
DNYJ=DNY(J)
DNZJ=DNZ(J)
U1J=U1(J)
U2J=U2(J)
U3J=U3(J)
V1J=V1(J)
V2J=V2(J)
W1J=W1(J)
```

C I=1

```
400 CONTINUE
IF (I.EQ.J) GO TO 250
CGXI=XM(I)
CGYI=YM(I)
CGZI=ZM(I)
DNXI= DNX(I)
DNYI= DNY(I)
DNZI= DNZ(I)
```

```
PB=DNXI*DLXJ+DNYI*DLYJ+DNZI*DLZJ
QB=DNXI*DUXJ+DNYI*DMYJ+DNZI*DMZJ
RB=DNXI*DNXJ+DNYI*DNYJ+DNZI*DNZJ
U =CGXI*DLXJ+CGYI*DLYJ+CGZI*DLZJ
V =CGXI*DMXJ+CGYI*DNYJ+CGZI*DNZJ
W =CGXI*DNXJ+CGYI*DNYJ+CGZI*DNZJ
```

```
P1=U1J-U
P2=U2J-U
P3=U3J-U
Q1=V1J-V
Q2=V2J-V
R1=W1J-W
```

```
RBR1=RB*R1
R1SQ=R1*R1
PM=(P1+P2+P3)*0.33333333
QM=(Q1+Q2+Q1)*0.33333333
AREA=0.5*ABS((P3-P1)*(Q2-Q1))
```

```

C
      GBAPR=AREA*(PB*PM+QB*QM+RBR1)/((PM*PM+QM*QM+R1SQ)**1.5)
      IF (ABS(GBAPR).GT.0.1E-02) GO TO 300
      GR(I,J)=GBAPR
      GO TO 200

C
      300 CONTINUE
      Q01=1.0/(Q2-Q1)
      Q02=U1*P2
      G1=(P2-P1)*Q01
      G2=(P2-P3)*Q01
      H1=(P1*Q2-QQ2)*Q01
      H2=(P3*Q2-QQ2)*Q01

C
      CALL GAUS10 (FX1,Q2,Q1,GS(I,J))
      GO TO 200

C
      250 GP(I,I)=TWOPI

C
      200 I=I+1
      IF (I.LE.NFACES) GO TO 400
      100 J=J+1
      IF (J.LE.NFACES) GO TO 500
      RETURN
      END

```

SUBROUTINE G13D (DEPTH)

GIBD COMPUTES THE INTERACTION TERMS BETWEEN FACES OF THE BODY AND FACES OF ITS IMAGE. THESE DEPEND ON THE BODY GEOMETRY AND ON THE BODY'S ATTITUDE AND POSITION W.R.T. THE SURFACE. THEY ARE INDEPENDENT OF THE SOURCE POSITION.

PULSE-TYPE BASIS FUNCTIONS ARE USED.

AN APPROXIMATE VALUE (GIAFR) IS FIRST CALCULATED. IF THE ABSOLUTE VALUE OF THIS IS GREATER THAN 0.001, THEN THE EXACT VALUE IS CALCULATED USING A 10-POINT GAUSS-LEGENDRE INTEGRATION FORMULA.

SUBROUTINE GAUS10 AND EXTERNAL FUNCTION FX1 ARE REQUIRED.

NFACES = NO. OF FACES
DEPTH = DEPTH TO CENTRE OF BODY COORDINATE SYSTEM.

C. T. BARNETT APRIL 1972

```

COMMON /BLOK2 / XM(75),YM(75),ZN(75)
COMMON /BLOK3 / DLX(75),DLY(75),DLZ(75),DMX(75),
&DMY(75),DNZ(75)
COMMON /BLOK4 / DMX(75),DMY(75),DNZ(75)
COMMON /BLOK5 / U1(75),U2(75),U3(75),V1(75),V2(75),W1(75)
COMMON /BLOK15/ GI(75,75)
COMMON /BFX1 / PR,CR,BBR1,P1SQ,G1,G2,H1,H2
COMMON /POLY / NFACES
EXTERNAL FX1

```

D2≈2.0★DEPTH
J=1

```

500 CONTINUE
DLXJ= DLX(J)
DLYJ= DLY(J)
DLZJ=-DLZ(J)
DMXJ= DMX(J)
DMYJ= DMY(J)
DNZJ=-DNZ(J)
DMXJ= DNX(J)
DNYJ= DNY(J)
DNZJ=-DNZ(J)
U1J=U1(J)
U2J=U2(J)
U3J=U3(J)
V1J=V1(J)
V2J=V2(J)
W1J=W1(J)

C
I=1
400 CONTINUE
CGXI=XM(I)
CGYI=YM(I)
CGZI=ZM(I)+D2
DNXI= DNX(I)
DNYI= DNY(I)
DNZI= DNZ(I)

C
PB=DNXI*Dlxj+DNYI*Dlyj+DNZI*DLZJ
QB=DNXI*DMXJ+DNYI*DNYJ+DNZI*DNZJ
RB=DNXI*DNXJ+DNYI*DNYJ+DNZI*DNZJ
U =CGXI*Dlxj+CGYI*Dlyj+CGZI*DLZJ
V =CGXI*DMXJ+CGYI*DNYJ+CGZI*DNZJ
W =CGXI*DNXJ+CGYI*DNYJ+CGZI*DNZJ

C
P1=U1J-U
P2=U2J-U
P3=U3J-U
Q1=V1J-V
Q2=V2J-V
R1=W1J-W

C
RRP1=RB*R1
R1SQ=R1*R1
PM=(P1+P2+P3)*0.33333333
QM=(Q1+Q2+Q1)*0.33333333
AREA=0.5*ABS((P3-P1)*(Q2-Q1))
GIAPR=AREA*((PB*PM+(B*QM+RBR1))/((P1*PM+QM*QM+R1SQ)**1.5))
IF (ABS(GIAPR).GT.0.1E-02) GO TO 300
GI(I,J)=GIAPR
GO TO 200

C
300 CONTINUE
QQ1=1.0/(Q2-Q1)
QQ2=Q1*P2
G1=(P2-P1)*QQ1
G2=(P2-P3)*QQ1
H1=(P1*Q2-QQ2)*QQ1
H2=(P3*Q2-QQ2)*QQ1

C
CALL GAUS10 (FX1,Q2,Q1,GI(I,J))

C

```

```

200 I=I+1
    IF (I.LE.NFACES) GO TO 400
100 J=J+1
    IF (J.LE.NFACES) GO TO 500
    RETURN
    END

```

C
C
C
SUBROUTINE H3D (X0,Y0,Z0,IDL)

C
C COMPUTES THE H ELEMENT FOR EACH FACE OF THE POLYHEDRON
FOR A PARTICULAR RECEIVER POSITION (POTENTIAL ELECTRODE).

C
C PULSE-TYPE BASIS FUNCTIONS ARE USED.

C
C AN APPROXIMATE VALUE IS FIRST CALCULATED (HAPR). IF THE ABSOLUTE
VALUE OF THIS IS GREATER THAN 0.02, THEN THE EXACT VALUE IS
CALCULATED USING A 10-POINT GAUSS-LEGENDRE INTEGRATION FORMULA.

C
C SUBROUTINE GAUS10 AND EXTERNAL FUNCTION FX2 ARE REQUIRED.

C
C NFACES = NO. OF FACES

C
C (-X0,-Y0,-Z0) ARE THE COORDINATES OF THE POTENTIAL ELECTRODE
W.R.T. THE CENTRE OF BODY COORDINATE SYSTEM.

C
C -----C. T. BARNETT-----APRIL 1972-----

C***MODIFIED BY JEFF DANIELS: 1976*****

```

COMMON /BLOK3 / DLX(75),DLY(75),DLZ(75),DMX(75),
&DMY(75),DMZ(75)
COMMON /BLOK4 / DNX(75),DNY(75),DNZ(75)
COMMON /BLOK5 / U1(75),U2(75),U3(75),V1(75),V2(75),W1(75)
COMMON /BLOK17/ H(75)
COMMON /BFX2 / R1SQ,G1,G2,H11,H2
COMMON /POLY / NFACES
      EXTERNAL FX2

```

400 CONTINUE

C

X0=X0

Y0=Y0

Z0=Z0

I=1

300 CONTINUE

U = X0*DLX(I)+Y0*DLY(I)+Z0*DLZ(I)

V = X0*DNX(I)+Y0*DMY(I)+Z0*DMZ(I)

W = X0*DNX(I)+Y0*DMY(I)+Z0*DMZ(I)

P1=U1(I)=U

P2=U2(I)=U

P3=U3(I)=U

Q1=V1(I)=V

Q2=V2(I)=V

R1=W1(I)=W

C

R1SQ=R1*R1

Pt=(P1+P2+P3)*0.33333333

Qn=(Q1+Q2+Q3)*0.33333333

AREA=0.5*ABS((P3-P1)*(Q2-Q1))

HAPR=AREA/SQRT(Pt*Pt+Qn*Qn+R1SQ)

IF (ABS(HAPR),GT,0.2E-01) GO TO 200

```

H(I)=HAPR
GO TO 100
C
200 CONTINUE
QQ1=1.0/(Q2-Q1)
QQ2=Q1*P2
G1=(P2-P1)*QQ1
G2=(P2-P3)*QQ1
H11=(P1*Q2-QQ2)*QQ1
H2=(P3*Q2-QQ2)*QQ1
C
CALL GAUS10 (FX2,Q1,Q2,H(I))
C
100 I=I+1
IF (I.LE.NFACES) GO TO 300
RETURN
END
C
C
C
C
SUBROUTINE F3D (X0,Y0,Z0,IDL,ZD)
C
C F3D COMPUTES THE F ELEMENT (SOURCE FUNCTION) FOR EACH FACE OF
C THE POLYHEDRON FOR A PARTICULAR SOURCE POSITION (CURRENT
C ELECTRODE) ON THE SURFACE.
C
C NFACES = NO. OF FACES
C (-X0,-Y0,-Z0) ARE THE COORDINATES OF THE CURRENT ELECTRODE
C W.R.T. THE CENTRE OF BODY COORDINATE SYSTEM.
C
C -----C.T.BARNETT-----APRIL 1972-----
C***MODIFIED BY JEFF DANIELS: 1976*****
C
COMMON /BLOK2 / XM(75),YM(75),ZN(75)
COMMON /BLOK4 / DNX(75),DNY(75),DNZ(75)
COMMON /BLOK7 / F(75)
COMMON /POLY / NFACES
DIMENSION F1(75)
C
X0=-X0
Y0=-Y0
Z0=Z0
II=1
DI=2.0
IF(IDFL.EQ.1) DI=1.0
400 I=1
200 CONTINUE
XI=XM(I)+X0
YI=YM(I)+Y0
ZI=ZN(I)+Z0
IF(IDFL.EQ.1.AND.II.EQ.1) ZJ=ZI-ZD
IF(IDFL.EQ.1.AND.II.NE.1) ZI=ZI+ZD
F(I)=DI*((XI*DNX(I)+YI*DNY(I)+ZI*DNZ(I))
& /((XI*XI+YI*YI+ZI*ZI)**1.5)
C
100 I=I+1
IF(I.LE.NFACES) GO TO 200
IF(IDFL.EQ.0) GO TO 700
IF(IL.EQ.0) GO TO 500

```

```

DO 800 IZ=1,NFACES
800 F1(IZ)=F(IZ)
II=0
GO TO 400
500 DO 600 IZ=1,NFACES
1002 FORMAT(5X,I5,2(2X,E12,6))
600 F(IZ)=(F(IZ)+F1(IZ))
700 CONTINUE
RETURN
END

```

C
C
C
C
C
C
C
SUBROUTINE GAUS10 (FUNCT,A,B,Y)

C
C THIS SUBROUTINE USES A 10-POINT GAUSS-LEGENDRE INTEGRATION
C FORMULA TO COMPUTE Y = INTEGRAL (FUNCT(X).DX)

C
C REFERENCES: THEORY AND PROBLEMS OF NUMERICAL ANALYSIS - F. SCHEID
C SCHAUM'S OUTLINE SERIES. PP. 134-137.

C
C HANDBOOK OF MATHEMATICAL CONSTANTS - Abramowitz and
C Stegun - P. 987, #25.4.30, and P. 916, TABLE 25.4

C
C SUBROUTINE PARAMETERS:

C
C FUNC = FUNCTION (EXTERNAL) TO BE INTEGRATED
C A = LOWER LIMIT OF INTEGRATION
C B = UPPER LIMIT OF INTEGRATION
C Y = SOLUTION RETURNED

C
C -----C,T,BARNETT-----APRIL 1972-----

C
C DIMENSION ZERO(5),COEFF(5)

C
C DATA ZERO / .14887434, .43339539, .67940957, .86506337, .97390653 /
C
C &, COEFF / .29552422, .26926672, .21908636, .14945135, .06667134 /

C
C Y=0.0

C
C C1=(B-A)*0.5

C
C C2=(A+B)*0.5

C
C DO 100 I=1,5

C
C C3=C1*ZERO(I)

C
C Y=Y+C1*COEFF(I)*(FUNCT(C2+C3)+FUNCT(C2-C3))

C
C 100 CONTINUE

C
C RETURN

C
C END

C
C FUNCTION FX1(Q)

C
C FX1 IS THE FUNCTION EXTERNAL TO BOTH G83D AND GI3D, WHICH IS
C CALLED BY SUBROUTINE GAUS10 TO COMPUTE THE EXACT VALUE OF THE
C BODY-BODY OR BODY-IMAGE INTERACTION TERMS.

C
C -----C,T,BARNETT-----APRIL 1972-----

C
C COMMON /BFX1 / PB,QB,RBR1,R1SQ,G1,G2,H1,H2

C
C FAC1=QB*Q+RBR1

```
FAC2=G2*Q+H2
FAC3=G1*Q+H1
FAC4=G*Q+R1SQ
FAC5=PB*FAC4
C
1   FX1=(FAC1*FAC2-FAC5)/(FAC4*SQRT(FAC2*FAC2+FAC4))
1   -(FAC1*FAC3-FAC5)/(FAC4*SQRT(FAC3*FAC3+FAC4))
```

```
C
RETURN
END
FUNCTION FX2 (Q)
```

```
C
C
C
C
C
FX2 IS THE FUNCTION EXTERNAL TO H3D, WHICH IS CALLED BY SUBROUTINE
GAUS10 TO COMPUTE THE EXACT VALUE OF THE H ELEMENT.
```

```
-----C.T.BARNETT-----APRIL 1972-----
```

```
COMMON /BFX2/ R1SQ,G1,G2,H1,H2
```

```
C
FAC2=G2*Q+H2
FAC3=G1*Q+H1
FAC4=G*Q+R1SQ
```

```
C
FX2= ALOG((FAC2+SQRT(FAC2*FAC2+FAC4))/(FAC3+SQRT(FAC3*FAC3+FAC4)))
```

```
C
RETURN
END
```

SUBROUTINE DECOMP

THIS SUBROUTINE USES GAUSSIAN ELIMINATION WITH PARTIAL PIVOTING
TO DECOMPOSE A SQUARE MATRIX INTO UPPER AND LOWER TRIANGULAR
FACTORS, AS A FIRST STEP TO SOLVING A SYSTEM OF LINEAR EQUATIONS.

REFERENCE - FORSYTHE, G., AND MOLER, C.B., 1967, COMPUTER SOLUTION
OF LINEAR ALGEBRAIC SYSTEMS; PRENTICE-HALL, P. 68-70.

A = THE MATRIX TO BE DECOMPOSED (NOTE: IN COMMON HERE).
N = NO. OF ROWS IN THE MATRIX (ALSO IN COMMON HERE).

IF A SINGULAR MATRIX IS ENCOUNTERED, ERROR MESSAGES ARE PRINTED
OUT, AND THE PROGRAM IS EXITED.

DEVICE SPECIFICATIONS -: OUTPUT (ERROR MESSAGES) = IOUT1

```
-----C.T.BARNETT-----APRIL 1972-----
```

```
COMMON /BLOK15/ A(75,75)
COMMON /DECSOL/ IPS(75),UL(75,75)
COMMON /ISPECS/ IN1,IN2,IOUT1,IOUT2,IOUT3
COMMON /POLY/ N
DIMENSION SCALES(75)
```

```

C
C      INITIALIZE UL,IPS AND SCALES
DO 102 I=1,N
  IPS(I)=I
  ROWNRM=0.0
  DO 101 J=1,N
    UL(I,J)=A(I,J)
    TEST=ABS(UL(I,J))
    IF (TEST.GT.ROWNRM) ROWNRM=TEST
101 CONTINUE
C
C      BOX 1 -- CHECK FOR SINGULARITY . . . . .
C
C      IF (ROWNRM.LT.0.1E-30) GO TO 301
C
C      SCALES(I)=1.0/ROWNRM
102 CONTINUE
C
C      GAUSSIAN ELIMINATION WITH PARTIAL PIVOTING
NM1=N-1
DO 106 K=1,NM1
  BIG=0.0
  DO 103 I=K,N
    IP=IPS(I)
    SIZE=ABS(UL(IP,K))*SCALES(IP)
    IF (SIZE.LE.BIG) GO TO 103
    BIG=SIZE
    IDXPIV=I
103 CONTINUE
C
C      BOX 2 -- CHECK FOR SINGULARITY . . . . .
C
C      IF (BIG.LT.0.1E-30) GO TO 302
C
C      IF (IDXPIV.EQ.K) GO TO 104
  J=IPS(K)
  IPS(K)=IPS(IDXPIV)
  IPS(IDXPIV)=J
104 KP=IPS(K)
  APIVOT=1.0/UL(KP,K)
  KP1=K+1
  DO 105 I=KP1,N
    IP=IPS(I)
    EM=-UL(IP,K)*APIVOT
    UL(IP,K)=-EM
    DO 105 J=KP1,N
      UL(IP,J)=UL(IP,J)+EM*UL(KP,J)
105 CONTINUE
106 CONTINUE
C
C      BOX 3 -- CHECK FOR SINGULARITY . . . . .
C
C      KP=IPS(N)
C      IF (ABS(UL(KP,N)).LT.0.1E-30) GO TO 303
C
C      RETURN
C
C      OR, PRINT ERROR MESSAGES AND EXIT
301 IBOX=1
  GO TO 304

```

```

302 IBOX=2
GO TO 304
303 IBOX=3
304 WRITE (IOUT1,12) IBOX
12 FORMAT (1H1,/,5X,'SINGULAR MATRIX DETECTED IN SUBROUTINE DECOMP. A
&T BOX!',I2,/,5X,'PROGRAM DISCONTINUED.')
CALL EXIT
END

```

C
C
C
C
C
C
SUBROUTINE SOLVE (B,X)

C
C
C
C
C THIS SUBROUTINE SOLVES A SET OF LINEAR SIMULTANEOUS EQUATIONS
C AFTER THEIR MATRIX HAS BEEN DECOMPOSED INTO UPPER AND LOWER
C TRIANGULAR FACTORS (SEE SUBROUTINE DECOMP).

C
C
C
C
C REFERENCE - FORSYTHE, G., AND MOLER, C.B., 1967, COMPUTER SOLUTION
C OF LINEAR ALGEBRAIC SYSTEMS; PRENTICE-HALL, P. 68-70.

C
C
C
C
C N = NO. OF ROWS IN THE MATRIX
B = THE CONSTANT VECTOR
X = THE SOLUTION VECTOR

C
C
C
C
C -----C.T.BARNETT-----APRIL 1972-----

C
COMMON /DECSOL/ IPS(75),UL(75,75)
COMMON /PDLY / N
DIMENSION B(75),X(75)

C
NP1=N+1

C
IP=IPS(1)
X(1)=B(IP)
DO 2 I=2,N
IP=IPS(I)
IM1=I-1
SUM=0.0
DO 1 J=1,IM1
1 SUM=SUM+UL(IP,J)*X(J)
2 X(I)=B(IP)-SUM

C
IP=IPS(N)
X(N)=X(N)/UL(IP,N)
DO 4 IBACK=2,N
I=NP1-IBACK
IP=IPS(I)
IP1=I+1
SUM=0.0
DO 3 J=IP1,N
3 SUM=SUM+UL(IP,J)*X(J)
4 X(I)=(X(I)-SUM)/UL(IP,I)

C
RETURN
END

SUBROUTINE NPAG3D (DIP,ZO,YO,AK,IPLOT,INDEX)

THIS SUBROUTINE PRINTS OUT THE HEADING INFORMATION ON PAGES
SUBSEQUENT TO PAGE 1 (SEE SUBROUTINE IPAG3D FOR PAGE 1 DETAILS).

DIP = DIP ANGLE TO WHICH BODY HAS BEEN ROTATED.
ZO = DEPTH TO ORIGIN OF BODY COORDINATE SYSTEM.
YO = Y-COORDINATE OF PROFILE ACROSS BODY.
AK = THE REFLECTION COEFFICIENT.

-----C.T.BARNETT-----APRIL 1972-----

COMMON /ISPECS/ IN1,IN2,IOUT1,IDUT2,IOUT3
COMMON /DATIME/ LABEL(16),IDATE(2),ITIME,NPAGES,IPAGE

WRITE (IOUT2,11) IDATE,ITIME
11 FORMAT (1H1,/,5X,'3-DIMENSIONAL IP AND RESISTIVITY MODELLING PROG
&RAM',//5X,1DATE: 12A5,10X,1TIME: 1,A5)
WRITE (IOUT2,12) LABEL
12 FORMAT (///,5X,'DESCRIPTION OF MODEL:1,4X,4A5,/30X,4A5,/30X,4A5,
&/30X,4A5)
WRITE (IOUT2,13) AK,ZO
13 FORMAT (//,5X,'REFLECTION COEFF. K =1,F8.5,/5X,'DEPTH =1,F8.2,
&//)

RETURN
END

SUBROUTINE ARAY3D (AK,BK,NDH,XPD,YPD,XD,YD,ZD,ZDA,ZDB,I6)

THIS SUBROUTINE COMPUTES THE RESPONSE TO THE VARIOUS 4-ELECTRODE
ARRAYS.

NFACES = NO. OF FACES TO THE POLYHEDRON
BK = FUNCTION OF THE REFLECTION COEFFICIENT REQUIRED TO COMPUTE
THE IP RESPONSE
XD,YD,ZD=SOURCE ELECTRODES
XPD,YPD= RECEIVER ELECTRODES
M=UPPER RECEIVER ELECTRODE
N=LOWER RECEIVER ELECTRODE
A=UPPER SOURCE ELECTRODE
B=LOWER SOURCE ELECTRODE

*****JEFF DANIELS,,FEB. 1975*****

*****USGS,DENVER*****

COMMON /BLOCK14/ HH(55,75),SS(75),TT(75),S(75),T(75)
COMMON /POLY / NFACES
COMMON/RESULT/ APRES(50),APIP(50),AR,PS,1R2,NSPA,ZPT
DOUBLE PRECISION DSUM,DTUM,RA,RT

```

C
K=IFIX(1./PS)
Y2=(YPD-YD)*(YPD-YD)
X2=(XPD-XD)*(XPD-XD)
IF(AR.GT.2.) GO TO 90
ZM=0.0
DO 110 I=1,(IR2-K)
XM=-11.5+FLOAT(I)*PS
XN=XM+1.0
XMM=(XM-XD)*(XM-XD)
XNN=(XN-XD)*(XN-XD)
AM=SQRT(XMM+Y2+ZDA*ZDA)
BM=SQRT(XMM+Y2+ZDB*ZDB)
AN=SQRT(XNN+Y2+ZDA*ZDA)
BN=SQRT(XNN+Y2+ZDB*ZDB)
IF(AR.NE.2) GO TO 81
AR=1.E+08
AN=1.E+08
81 CONTINUE
DSUM=0.0D 00
DTUM=0.0D 00
163 FORMAT(1X,'HH(I,1)',E12.6,'HH(I+1,1)',E12.6,'SS(1)',E12.6,
&'S(1)',E12.6)
DO 120 J=1,NFACES
DSUM=DSUM +(SS(J)-S(J))*(HH(I,J)-HH((I+K),J))
120 DTUM=DTUM+(TT(J)-T(J))*(HH(I,J)-HH((I+K),J))
RQ=1./AM-1./BM-1./AN+1./BN
APRES(I)=1.0+DSUM/((1./AM-1./BM-1./AN+1./BN))
APIP(I)=(1.-AK*AK)*DTUM/(2.*((1./AM-1./BM-1./AN+1./BN))/APRES(I))
110 CONTINUE
GO TO 100
90 CONTINUE
IF(AR.NE.4) GO TO 70
DO 140 I=1,(IR2-K)
ZN=FLOAT(I)*PS
ZN=ZN+1.0
AM=1.E+08
AN=1.E+08
BM=SQRT(X2+Y2+ZN*ZN)
BN=SQRT(X2+Y2+ZN*ZN)
DSUM=0.0D+00
DTUM=0.0D+00
DO 150 J=1,NFACES
DSUM=DSUM +(SS(J)-S(J))*(HH(I,J)-HH((I+K),J))
150 DTUM=DTUM+(TT(J)-T(J))*(HH(I,J)-HH((I+K),J))
RQ=1./BN-1./BM
APRES(I)=1.+DSUM/RQ
140 APIP(I)=(1.-AK*AK)*DTUM/(2.*RQ)/APRES(I)
GO TO 100
70 CONTINUE
IF(AR.EQ.6.) GO TO 92
IF(AR.EQ.7.) GO TO 93
91 ZM=0.0
IS1=1
IS2=IR2-K
GO TO 94
92 ZM=ZDA
ZN=ZDB
IS1=1
IS2=1

```

```

        GO TO 94
93 X2=0.0
Y2=0.0
ZM=FLOAT(NSPA)+ZDB
ZN=ZM+1
IS1=IFIX(ZN/PS)
IS2=IS1
94 DO 105 I=IS1,IS2
IF(AR,GE,6) GO TO 95
I1=I
ZM=FLOAT(I)*PS
ZN=ZM+1.
95 CONTINUE
IF(AR,NE,7) GO TO 96
X2=0.0
Y2=0.0
96 CONTINUE
AM=SQRT(X2+Y2+(ZN-ZDA)*(ZN-ZDA))
ATM=SQRT(X2+Y2+(ZN+ZDA)*(ZN+ZDA))
BM=SQRT(X2+Y2+(ZN-ZDB)*(ZN-ZDB))
BTM=SQRT(X2+Y2+(ZN+ZDB)*(ZN+ZDB))
AN=SQRT(X2+Y2+(ZN-ZDA)*(ZN-ZDA))
ATN=SQRT(X2+Y2+(ZN+ZDA)*(ZN+ZDA))
BN=SQRT(X2+Y2+(ZN-ZDB)*(ZN-ZDB))
BTN=SQRT(X2+Y2+(ZN+ZDB)*(ZN+ZDB))
IF(AR,NE,3) GO TO 64
AM=1.0E+08
ATM=1.0E+08
AN=1.0E+08
ATN=1.0E+08
64 CONTINUE
RA=DBLE(1./AM)-DBLE(1./BM)-DBLE(1./AN)+DBLE(1./BN)
RT=DBLE(1./ATM)-DBLE(1./BTM)-DBLE(1./ATH)+DBLE(1./BTN)
DSUM=0.0D 00
DTUM=0.0D 00
IF(AR,EQ,6) I1=I6
IF(AR,EQ,7) I1=IFIX(ZM/PS)
DO 130 J=1,NFACES
DSUM=DSUM+DBLE(SS(J)-S(J))*DBLE(HH(I1,J)-HH((I1+K),J))
130 DTUM=DTUM+DBLE(TT(J)-T(J))*DBLE(HH(I1,J)-HH((I1+K),J))
IF(AR,EQ,3.OR.AR,EQ,5) GO TO 107
APRES(I6)= 1.0 +DSUM/(RA+RT)
APIP(I6)=(1.0-AK*AK)*DTUM/(2.0*(RA+RT))/APRES(I6)
GO TO 105
107 APRES(I)=1.+DSUM/(RA+RT)
APIP(I)=(1.-AK*AK)*DTUM/(2.0*(RA+RT))/APRES(I)
105 CONTINUE
100 CONTINUE
RETURN
C
END
C
C
C
C
SUBROUTINE OUTPUT (IDH,IPILOT,XPD,YPD,XD,YD,ZD,ZDA,ZDB)
C
C
C
C
-----C.T.BARNETT-----APRIL 1972-----
C*****MODIFIED BY JEFF DANIELS, 1975*****
```

```

C
COMMON /ISPECS/ IN1,IN2,IOUT1,IOUT2,IOUT3
COMMON/RESULT/ APRES(50),APIP(50),AR,PS,IR2,NSPA,ZPT
C
12 FORMAT (4E15.8)
C
C
C THIS SECTION OUTPUTS RESULTS FROM THE DOWNHOLE CONFIGURATION
C
K=IFIX(1./PS)
IF(AR.EQ.1.) GO TO 100
IF(AR.EQ.2.) GO TO 200
IF(AR.EQ.3.) GO TO 300
IF(AR.EQ.4.) GO TO 400
IF(AR.EQ.5.) GO TO 500
IF(AR.EQ.6.) GO TO 600
IF(AR.EQ.7.) GO TO 700
C HOLE-BIPOLE SOURCE, SURFACE BIPOLE RECEIVER
100 WRITE(IOUT2,101)
      WRITE(IOUT2,102) ZDA,ZDB,XD,YD
      WRITE(IOUT2,106)YPD
      WRITE(IOUT2,103)
      DO 104 IX=1,(IR2-K)
      XP=-11.0+FLOAT(IX)*PS
104  WRITE(IOUT2,105) XP,APRES(IX),APIP(IX)
      GO TO 1000
*****
C BURIED POLE SOURCE, SURFACE BIPOLE RECEIVER
*****
200 WRITE(IOUT2,201)
      WRITE(IOUT2,102) ZDA,ZDB,XD,YD
      WRITE(IOUT2,106) YPD
      WRITE(IOUT2,103)
      DO 204 IX=1,(IR2-K)
      XP=-11.0+FLOAT(IX)*PS
204  WRITE(IOUT2,105)XP,APRES(IX),APIP(IX)
      GO TO 1000
*****
C BURIED POLE SOURCE, BURIED BIPOLE RECEIVER
*****
300 WRITE(IOUT2,301)
      WRITE(IOUT2,102)ZDA,ZDB,XD,YD
      WRITE(IOUT2,302) XPD,YPD
      WRITE(IOUT2,303)
      DO 304 IX=1,(IR2-K)
      ZP=0.5+FLOAT(IX)*PS
304  WRITE(IOUT2,105) ZP,APRES(IX),APIP(IX)
      GO TO 1000
*****
C SURFACE POLE SOURCE, BURIED BIPOLE RECEIVER
*****
400 WRITE(IOUT2,401)
      WRITE(IOUT2,603) XD,YD
      WRITE(IOUT2,302) XPD,YPD
      WRITE(IOUT2,303)
      DO 505 IX=1,(IR2-K)
      ZP=0.5+FLOAT(IX)*PS
505  WRITE(IOUT2,105) ZP,APRES(IX),APIP(IX)
      GO TO 1000

```

```

*****
C BURIED BIPOLE FIXED SOURCE, BURIED BIPOLE RECEIVER
*****
500 WRITE(IOUT2,501)
    WRITE(IOUT2,102) ZDA,ZDR,XD,YD
    WRITE(IOUT2,302) XPD,YPD
    WRITE(IOUT2,303)
    DO 504 IX=1,(IR2-1)
    ZP=0.5+FLOAT(IX)*PS
504 WRITE(IOUT2,105) ZP,APRES(IX),APIP(IX)
    GO TO 1000
*****
C BURIED BIPOLE MOVING SOURCE,BURIED BIPOLE RECEIVER
*****
600 WRITE(IOUT2,601)
    WRITE(IOUT2,603) XD,YD
    WRITE(IOUT2,302) XPD,YPD
    WRITE(IOUT2,303)
    DO 602 IQ=1,(IR2-K)
    ZP=0.5+FLOAT(IQ)*PS
602 WRITE(IOUT2,105) ZP,APRES(IQ),APIP(IQ)
    GO TO 1000
*****
C SINGLE HOLE BIPOLE-BIPOLE CONFIGURATION
*****
700 WRITE(ICUT2,701)
    WRITE(IOUT2,703) XD,YD
    WRITE(IOUT2,303)
    IRP1=IR2-IFIX(1/PS+NSPA/PS)
    DO 702 IQ=1,IPPI
    ZP=FLOAT(NSPA)/2+PS*FLOAT(IQ)+1.
702 WRITE(IOUT2,105) ZP,APRES(IQ),APIP(IQ)
1000 CONTINUE
105 FORMAT(2X,F8.3,2(2X,E12.6))
101 FORMAT(2X,30('!'),/,2X,'BURIED BIPOLE SOURCE,SURFACE
&BIPOLE RECEIVER!',/,30('!'),/)
201 FORMAT(2X,30('!'),/,2X,'BURIED POLE SOURCE,SURFACE
&BIPOLE RECEIVER!',/,30('!'),/)
301 FORMAT(2X,30('!'),/,2X,'BURIED POLE SOURCE, BURIED BIPOLE
&RECEIVER!',/,30('!'),/)
401 FORMAT(2X,30('!'),/,2X,'SURFACE POLE SOURCE,BURIED BIPOLE
& RECEIVER!',/,30('!'),/)
501 FORMAT(2X,30('!'),/,2X,'BURIED BIPOLE FIXED SOURCE, BURIED
&BIPOLE RECEIVER!',/,30('!'),/)
601 FORMAT(2X,30('!'),/,2X,'BURIED BIPOLE MOVING SOURCE,BURIED BIPOLE
&RECEIVER!',/,30('!'),/)
701 FORMAT(2X,30('!'),/,2X,' SINGLE HOLE,BIPOLE-BIPOLE!',/,30('!'),/)
102 FORMAT(2X,'UPPER SOURCE=',F10.3,/,2X,'LOWER SOURCE=',F10.3,
/,2X,'X-SOURCE=',F10.3,/,2X,'Y-SOURCE=',F10.3,/)
106 FORMAT(2X,'Y-PROFILE=',F10.3,/)
103 FORMAT(2X,'X-RECEIVER!',2X,'APPARENT',5X,'APPARENT',/,2X,
&'POSITION',4X,'RESISTIVITY',2X,'POLARIZABILITY',/)
302 FORMAT(2X,'X-RECEIVER=',F10.3,/,2X,'Y-RECEIVER=',F10.3,/)
303 FORMAT(5X,'RECEIVER',2X,'APPARENT',5X,'APPARENT',/,5X,'DEPTH',
&6X,'RESISTIVITY',2X,'POLARIZABILITY',/)
603 FORMAT(2X,'X-SOURCE=',F10.3,/,2X,'Y-SOURCE=',F10.3,/)
703 FORMAT(2X,'X-POSITION OF HOLE=',F10.3,/,2X,'Y-
&POSITION OF HOLE=',F10.3,/)
    RETURN
END

```

```

C***** SUBROUTINE BODY3D *****
C
C THIS SUBPROGRAM CALCULATES POINTS FOR 3-DIMENSIONAL BODIES.
C THE BASIC BODY IS AN ELLIPSOID. A SPHERE MAY BE
C GENERATED BY MAKING A=B=C. OUTPUT IS IN THE FORM OF CONTOURS
C OF (X,Y) VALUES AT Z VALUES. THE OUTPUT IS DESIGNED TO BE USED
C WITH IP3DDH, THE CENTER OF THE BODY IS ZERO.
C
C PARAMETERS:
C A=MAXIMUM X-DIRECTION OF BODY(HALF-WIDTH)
C B=MAXIMUM Y-DIRECTION OF BODY (HALF-WIDTH)
C C=MAXIMUM Z-DIRECTION OF BODY (HALF-WIDTH)
C DEL=SEPERATION OF Z POINTS(SEPARATION OF CONTOURS)
C = (C*2)/4
C PARAMETERS A,B, AND C ARE READ FROM DISK,DEL IS COMPUTED.
C
C JEFF DANIELS
C USGS DENVER
C MARCH 1, 1975
C
C SUBROUTINE BODY3D
C DIMENSION X(100),Y(100),VX(100),VY(100)
C ,VXS(10),VYS(10)
C COMMON/BLOK12/NC1,NV
C COMMON/BODY/VXA(100),VYA(100),Z(5)
C IN=13
C READ(IN,6)A,B,C
C DEL=(C*2.)/4.
C NC=4
C ICA=0
C IC=0
C DO 21 I=1,NC+1
C M=I-1
C Z(I)=C+M*DEL
C XM=SQRT(A*A*(1.-Z(I)*Z(I)/(C*C)))
C IF(XM,EQ.,0.0) MM=1
C IF(XM,EO.,0.0) GO TO 42
C DEL1=XM/3
C MM=3
C 42 DO 20 J=1,MM
C L=J-1
C X(J)=XM-L*DEL1
C Y(J)=SQRT(B*B*(1.-Z(I)*Z(I)/(C*C))-X(J)*X(J)/(A*A)))
C IC=IC+1
C VX(IC)=X(J)
C VY(IC)=Y(J)
C 20 CONTINUE
C IF(I,EQ.,1,OR,I.EQ.(NC+1)) GO TO 24
C IC=IC+1
C VX(IC)=0.0
C VY(IC)=SQRT(B*B*(1.-Z(I)*Z(I)/(C*C)))
C DQ 22 K=1,MM
C X(MM+K)=-X(MM-K+1)
C Y(MM+K)=Y(MM+1-K)
C IC=IC+1
C VX(IC)=X(MM+K)
C VY(IC)=Y(MM+K)
C 22 CONTINUE
C MM=M+MM-1

```

```

IF(ICA.EQ.0) KG=3
IF(ICA.GT.0) KG=2
DO 23 KB=1,MM+LL
IC=IC+1
VX(IC)=VX(MMM+ICA-KB+KG)
VY(IC)=-VY(MMM+ICA-KB+KG)
23 CONTINUE
ICA=IC
24 IF(MM.EQ.1.OR.MM.EQ.(NC+1)) MMQ=MM+1
21 CONTINUE
30 CONTINUE
NV=(IC-2)/(NC-1)
NC1=NC-1
NCC=NC1*NV+2
VXA(1)=VX(1)
VYA(1)=VY(1)
I6=2
DO 700 I4=1,NC1
IQ=I4+1
IST=I4
DO 800 I7=1,IST
VXS(I7)=VX(I6+I7-1)
800 VYS(I7)=VY(I6+I7-1)
DO 600 I5=IQ,NV
VXA(I6)=VX(I6+IST)
VYA(I6)=VY(I6+IST)
600 I6=I6+1
DO 900 I9=1,IST
VXA(I6)=VXS(I9)
VYA(I6)=VYS(I9)
900 I6=I6+1
700 CONTINUE
VXA(IC)=VX(IC)
VYA(IC)=VY(IC)
WRITE(15,201)(Z(IL),IL=1,5)
WRITE(15,201)(VXA(IL),VYA(IL),IL=1,IC)
201 FORMAT(2(2X,E12.6))
6 FORMAT(3F)
RETURN
END

```